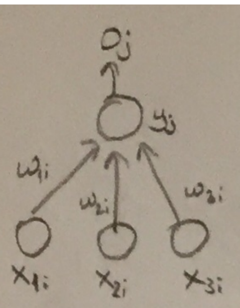


$$y' = \sum w_i x_i$$

$\hat{y}$: Desired output

y' : Calculated output

$$E = \frac{1}{2}(\hat{y} - y')^2$$

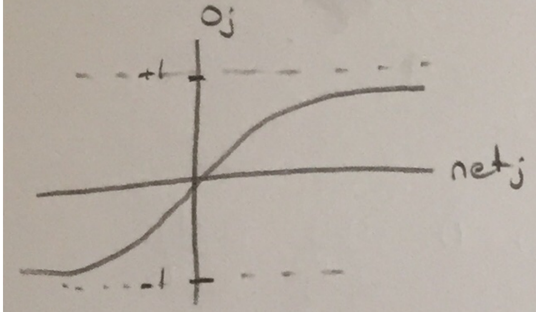
$$\Delta w_i = -\alpha . \frac{\partial E}{\partial w_i} = -\alpha . (\hat{y} - y') . (-1) . \frac{dy'}{dw_i}$$

$$= \alpha . (\hat{y} - y') . x_i$$

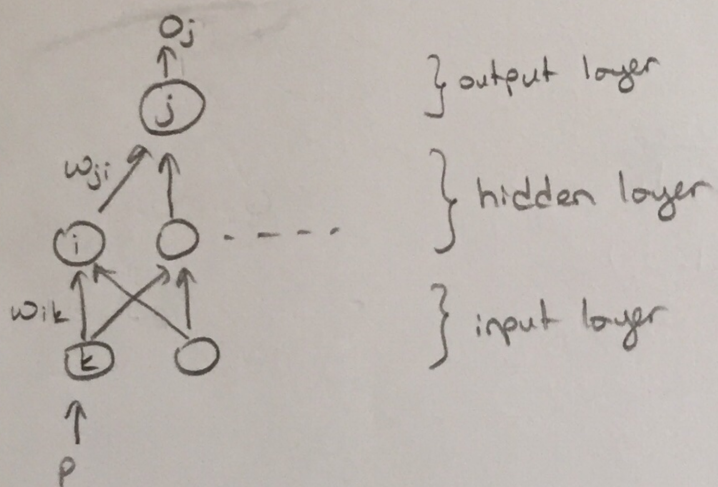
$$net_j = \sum_i w_j . x_i \qquad o_j = \begin{cases} 1 & \text{if } net_j \geq 0 \\ 0 & \text{otherwise} \end{cases}$$

For normalized or precise output, we are going to use squashing function

$$f(x) = \tanh(x) \implies o_j = \tanh(net_j)$$



$$\implies \frac{\partial f(x)}{\partial x} = 1 - f^2(x)$$



t_{pj} = desired output for node j

$$Error_{on_j} = \sum_j \frac{1}{2}(t_{pj} - o_j)^2$$

$$net_j = \sum_i w_{ji} . x_i \qquad net_i = \sum_k w_{ik} . x_k$$

$$O_j = \tanh(net_j) \qquad O_i = \tanh(net_i)$$

$$x_i = O_i$$

* We want to calculate $\frac{\partial E}{\partial w}$, the rate of change of error with respect to the given connective weight, so we can minimize it.

$$\frac{\partial E}{\partial w_{ji}} = \frac{\partial E}{\partial o_j} \cdot \frac{\partial o_j}{\partial net_j} \cdot \frac{\partial net_j}{\partial w_j}$$

$$= (t_{pj} - o_j).(-1).(1-o_j^2).x_i$$

$$\Delta w_{ji} = -\alpha . \frac{\partial E}{\partial w_{ji}}$$

$$= +\alpha . \underbrace{(t_{pj} - o_j).(+1).(1-o_j^2)}_{\delta_{pj}} . x_i$$

$$\delta_{pj} = \frac{-\partial E_p}{\partial net_j}$$

$$\Delta w_{ji} = \alpha . \delta_{pj} . x_i$$

$$\Delta w_{ik} = -\alpha . \frac{\partial E}{\partial w_{ik}} = -\alpha . \underbrace{\frac{\partial E}{\partial o_i} . \frac{\partial o_i}{\partial net_i}}_{-\delta_{pi}} . \frac{\partial net_i}{\partial w_{ik}}$$

$$\Delta w_{ik} = \alpha . \delta_{pi} . x_k$$

$$\delta_{pi} = \sum_j (t_{pj} - o_j).(-1). \frac{\partial o_j}{\partial net_i} = \sum_j (t_{pj} - o_j).(-1). \frac{\partial o_j}{\partial net_j} . \frac{\partial net_j}{\partial net_i}$$

$$\sum_j (t_{pj} - o_j).(-1). \frac{\partial o_j}{\partial net_j} . \frac{\partial net_j}{\partial o_i} . \frac{\partial o_i}{\partial net_i}$$

$$= \left(\sum_j \underbrace{(t_{pj} - o_j).(-1).(1-o_j^2)}_{-\delta_{pj}} . w_{ji} \right) (1-o_i^2)$$

$$\delta_{pi} = (1-o_i^2) . \sum_j -\delta_{pj} . w_{ji}$$