

Stack Use



What is a stack?

- A **stack** is a Last In, First Out (LIFO) data structure
- Anything added to the stack goes on the “top” of the stack
- Anything removed from the stack is taken from the “top” of the stack
- Things are removed in the reverse order from that in which they were inserted

Constructing a stack

- To use Java's built-in stacks, you need either of
 - `import java.util.*;`
 - `import java.util.Stack;`
- There is just one stack constructor:
`Stack<E> stack = new Stack<E>();`
- `E` is the type of element (for example, `String`) that you intend to put on the stack
 - To get the old (pre-Java 5) behavior, where objects of any kind can be put on the stack, use `<?>`

Fundamental stack operations

`oldItem = stack.push(item)`

- Adds the item (of type `E`) to the top of the stack; the item pushed is also returned as the value of `push`

`item = stack.pop();`

- Removes the `E` at the top of the stack and returns it

`item = stack.peek();`

- Returns the top `E` of the stack but does not remove it from the stack

`stack.empty()`

- Returns `true` if there is nothing in the stack

Additional stack operation

```
int i = stack.search(item);
```

- Returns the *1-based* position of the element on the stack. That is, the top element is at position **1**, the next element is at position **2**, and so on.
- Returns **-1** if the element is not on the stack
- This is not a “conventional” stack operation, but is sometimes useful

Inheritance vs. composition

- This is **inheritance**: `class X extends class Y { ... }`
 - `X` inherits all the (non-private) methods and variables of `Y`
 - This is appropriate if `X` is a *kind of* `Y`, but not otherwise
 - Because you get *all* of `Y`, whether it's appropriate for `X` or not
 - Inheritance is often overused
- This is **composition**: `class X { Y myYvariable; ... }`
 - To make a method, say `int m(int a)`, of class `Y` available to objects of class `X`, you use **delegation**:
`class X { ...; int m(int a) { return myYVariable.m(a); } ... }`
 - Similarly, class `X` can have getters and setters that refer to variables of the object `myYVariable`
 - Composition is appropriate if class `X` *uses* class `Y`, but isn't a *kind of* class `Y`
- If in doubt, use composition rather than inheritance

Stack ancestry

- The **Stack** class extends (inherits from) the **Vector** class
 - Hence, anything you can do with a **Vector**, you can also do with a **Stack**
 - For example, you can index into it, add elements to and remove elements from the middle, etc.
 - This is not how stacks should be used!
 - If you want **Vector** operations, use a **Vector**--the **Stack** methods give you conventional names for stack operations, but no new capabilities
 - A “stack” is a very specific data structure, defined by the preceding operations; it just happens to be *implemented* by extending **Vector**
 - Even Sun gets things wrong sometimes
- The **Vector** class implements the **Collection** interface
 - Hence, anything you can do with a **Collection**, you can also do with a **Stack**
 - The most useful operation this gives you is **toArray()**

Some uses of stacks

- Stacks are used for:
 - Any sort of nesting (such as parentheses)
 - Evaluating arithmetic expressions (and other sorts of expression)
 - Implementing function or method calls
 - Keeping track of previous choices (as in backtracking)
 - Keeping track of choices yet to be made (as in creating a maze)

A balancing act

- $([](\{()\}[()]))$ is balanced; $([](\{()\}[()]))$ is not
- Simple counting is not enough to check balance
- You can do it with a stack: going left to right,
 - If you see a $($, $[$, or $\{$, push it on the stack
 - If you see a $)$, $]$, or $\}$, pop the stack and check whether you got the corresponding $($, $[$, or $\{$
 - When you reach the end, check that the stack is empty

Expression evaluation

- Almost all higher-level languages let you evaluate expressions, such as $3*x+y$ or $m=m+1$
- The simplest case of an expression is one number (such as 3) or one variable name (such as x)
 - These *are* expressions
- In many languages, $=$ is considered to be an operator
 - Its value is (typically) the value of the left-hand side, after the assignment has occurred
- Situations sometimes arise where you want to evaluate expressions yourself, without benefit of a compiler

Performing calculations

- To evaluate an expression, such as $1+2*3+4$, you need *two* stacks: one for operands (numbers), the other for operators: going left to right,
 - If you see a number, push it on the number stack
 - If you see an operator,
 - **While** the top of the operator stack holds an operator of equal or higher precedence:
 - pop the old operator
 - pop the top two values from the number stack and apply the old operator to them
 - push the result on the number stack
 - push the new operator on the operator stack
 - At the end, perform any remaining operations

Example: $1+2*3+4$

- 1 : push 1 on number stack
- + : push + on op stack
- 2 : push 2 on number stack
- * : because * has higher precedence than +, push * onto op stack
- 3 : push 3 onto number stack
- + : because + has lower precedence than *:
 - pop 3, 2, and *
 - compute $2*3=6$, and push 6 onto number stack
 - push + onto op stack
- 4 : push 4 onto number stack
- end : pop 4, 6 and +, compute $6+4=10$, push 10; pop 10, 1, and +, compute $1+10=11$, push 11
- 11 (at the top of the stack) is the answer

Handling parentheses

- When you see a left parenthesis, (, treat it as a low-priority operator, and just put it on the operator stack
- When you see a right parenthesis,), perform all the operations on the operator stack until you reach the corresponding left parenthesis; then remove the left parenthesis

Handling variables

- There are two ways to handle variables in an expression:
 - When you encounter the variable, look up its value, and put its value on the operand (number) stack
 - This simplifies working with the stack, since everything on it is a number
 - When you encounter a variable, put the variable itself on the stack; only look up its value later, when you need it
 - This allows you to have embedded assignments, such as $12 + (x = 5) * x$

Handling the = operator

- The assignment operator is just another operator
 - It has a lower precedence than the arithmetic operators
 - It should have a higher precedence than (
- To evaluate the = operator:
 - Evaluate the right-hand side (this will already have been done, if = has a low precedence)
 - Store the value of the right-hand side into the variable on the left-hand side
 - You can only do this if your stack contains variables as well as numbers
 - Push the value onto the stack

At the end

- Two things result in multiple special cases
 - You frequently need to compare the priority of the current operator with the priority of the operator at the top of the stack—but the stack may be empty
 - Earlier, I said: “At the end, perform any remaining operations”
- There is a simple way to avoid these special cases
 - Invent a new “operator,” say, \$, and push it on the stack initially
 - Give this operator the lowest possible priority
 - To “apply” this operator, just quit—you’re done

Some things that can go wrong

- The expression may be ill-formed:

2 + 3 +

- When you go to evaluate the second +, there won't be two numbers on the stack

1 2 + 3

- When you are done evaluating the expression, you have more than one number on the stack

(2 + 3

- You have an unmatched (on the stack

2 + 3)

- You can't find a matching (on the stack

- The expression may use a variable that has not been assigned a value

Types of storage

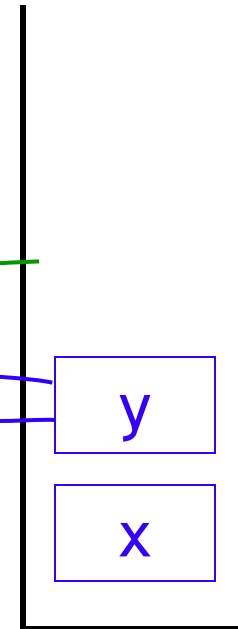
- In almost all languages (including Java), data is stored in two different ways:
 - Temporary variables—parameters and local variables of a method—are stored in a *stack*
 - These values are popped off the stack when the method returns
 - The value returned from a method is also temporary, and is put on the stack when the method returns, and removed again by the calling program
 - More permanent variables—objects and their instance variables and class variables—are kept in a *heap*
 - They remain on the heap until they are “freed” by the programmer (C, C++) or garbage collected (Java)

Stacks in Java

- Stacks are used for local variables (including parameters)

```
void methodA() {  
    int x, y; // puts x, y on stack  
    y = 0;  
    methodB();  
    y++;  
}
```

```
void methodB() {  
    int y, z; // puts y, z on stack  
    y = 5;  
    return; // removes y, z  
}
```



Recursion

- A **recursive definition** is when something is defined *partly* in terms of itself
- Here's the mathematical definition of **factorial**:

$$\text{factorial}(n) = \begin{cases} 1, & \text{if } n \leq 1 \\ n * \text{factorial}(n - 1) & \text{otherwise} \end{cases}$$

- Here's the programming definition of factorial:

```
static int factorial(int n) {  
    if (n <= 1) return 1;  
    else return n * factorial(n - 1);  
}
```

Supporting recursion

```
static int factorial(int n) {  
    if (n <= 1) return 1;  
    else return n * factorial(n - 1);  
}
```

- If you call `x = factorial(3)`, this enters the factorial method with `n=3` on the stack
- | `factorial` calls itself, putting `n=2` on the stack
- | | `factorial` calls itself, putting `n=1` on the stack
- | | `factorial` returns `1`
- | `factorial` has `n=2`, computes and returns `2*1 = 2`
- `factorial` has `n=3`, computes and returns `3*2 = 6`

Factorial (animation 1)

- `x = factorial(3)`

3 is put on stack as n

- `static int factorial(int n) { //n=3`

`int r = 1;` r is put on stack with value 1

`if (n <= 1) return r;`

`else {`

`r = n * factorial(n - 1);`

`return r;`

`}`

`}`

All references to r use this r

All references to n use this n

Now we recur with 2...

r=1

n=3

Factorial (animation 2)

- $r = n * \text{factorial}(n - 1);$

2 is put on stack as n

- `static int factorial(int n) { // n=2`

`int r = 1;` r is put on stack with value 1

`if (n <= 1) return r;`

`else {`

`r = n * factorial(n - 1);`

`return r;`

`}`

`}`

Now using this r

And this n

Now we recur with 1...

r=1

n=2

r=1

n=3

Factorial (animation 3)

- $r = n * \text{factorial}(n - 1);$

1 is put on stack as n

- ```
static int factorial(int n) {
 int r = 1;
 if (n <= 1) return r;
 else {
 r = n * factorial(n - 1);
 return r;
 }
}
```

Now using this r  
And this n

r=1

n=1

r=1

n=2

r=1

n=3

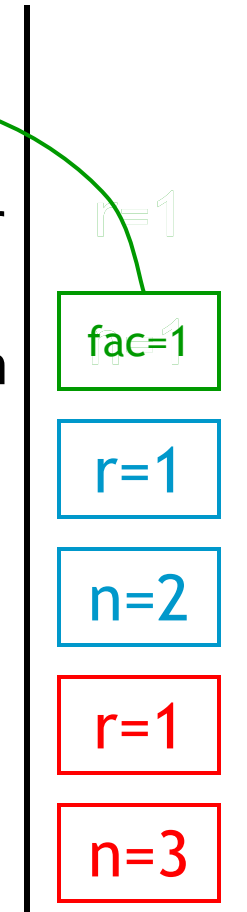
Now we pop r and n  
off the stack and return  
1 as factorial(1)

# Factorial (animation 4)

- $r = n * \text{factorial}(n - 1);$

- ```
static int factorial(int n) {  
    int r = 1;  
    if (n <= 1) return r;  
    else {  
        r = n * factorial(n - 1);  
        return r;  
    }  
}
```

Now we pop r and n
off the stack and return
1 as factorial(1)



Factorial (animation 5)

- $r = n * \text{factorial}(n - 1);$

- ```
static int factorial(int n) {
 int r = 1;
 if (n <= 1) return r;
 else {
 r = n * factorial(n - 1);
 return r;
 }
}
```

Now using this r

And  
this n

fac=2

r=1

n=3

2 \* 1 is 2;  
Pop r and n;

Return 2

# Factorial (animation 6)

- x = factorial(3)

- static int factorial(int n) {  
    int r = 1;  
    if (n <= 1) return r;  
    else {  
        r = n \* factorial(n - 1);  
        return r;  
    }  
}

3 \* 2 is 6;

Pop r and n;

Return 6

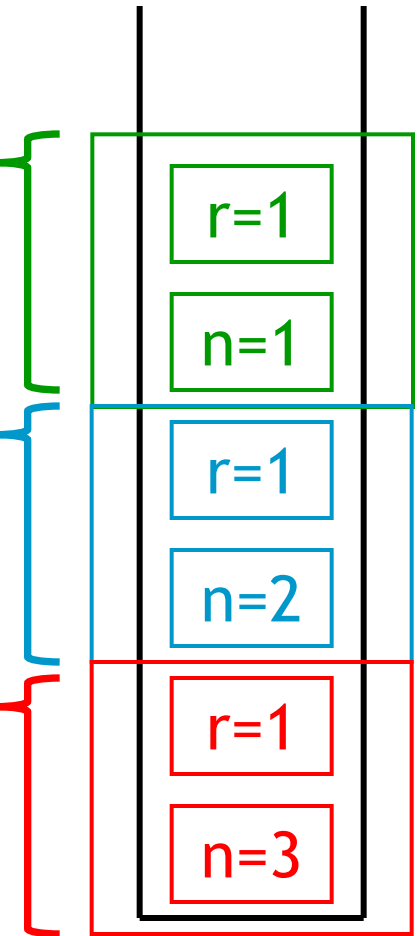
Now using this r

And  
this n

r=1  
fac=  
n=3  
6

# Stack frames

- Rather than pop variables off the stack one at a time, they are usually organized into **stack frames**
- Each frame provides a set of variables and their values
- This allows variables to be popped off all at once
- There are several different ways stack frames can be implemented



# Summary

- Stacks are useful for working with any nested structure, such as:
  - Arithmetic expressions
  - Nested statements in a programming language
  - *Any* sort of nested data

# The End

