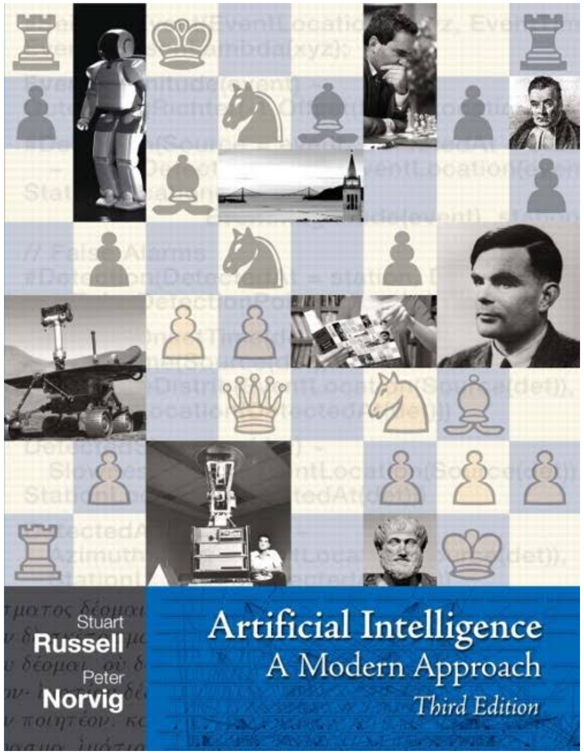
Sources & credits



- Course notes:
  - Russell and Norvig:
    - http://aima.cs.berkeley.edu/
  - Levent Akin, Pinar Yolum and Albert Ali Salah
    - http://www.cmpe.boun.edu.tr/~akin/
    - http://mas.cmpe.boun.edu.tr/wiki/doku.php?id=courses:cmpe540:info
- Projects and content:
  - UC Berkeley CS188 Intro to AI
    - https://inst.eecs.berkeley.edu/~cs188/fa11/assignments.html

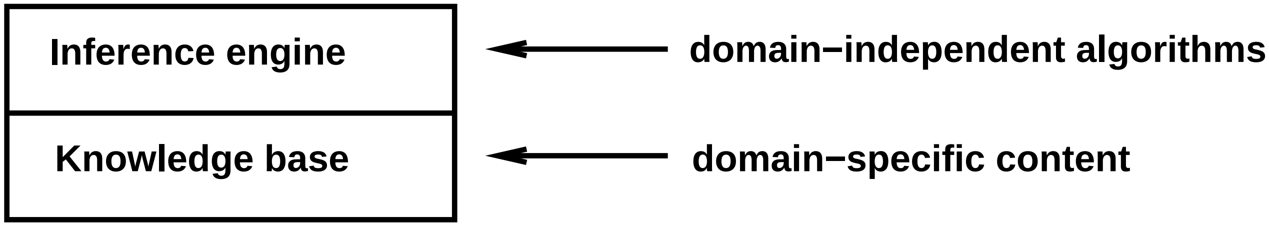
Outline

- ◊ Knowledge-based agents

flexible: accept new tasks in the form of explicitly described goals
partial obs: infer hidden aspects of the current state
powerful: combine and recombine information
generic: knowledge is expressed in general forms

- ◊ Wumpus world
- ◊ Logic in general—models and entailment
- ◊ Propositional (Boolean) logic
- ◊ Equivalence, validity, satisfiability
- ◊ Inference rules and theorem proving
  - – forward chaining
  - – backward chaining
  - – resolution

Knowledge bases



Knowledge base = set of sentences in a **formal** language

Declarative approach to building an agent (or other system):
TELL it what it needs to know

Then it can ASK itself what to do—answers should follow from the KB

Agents can be viewed at the knowledge level
i.e., **what they know**, regardless of how implemented

Or at the implementation level
i.e., data structures in KB and algorithms that manipulate them

Declarative approach: in the form of sentences: adv?

Procedural approach: program code. minimizing the representation and reasoning: adv?

A simple knowledge-based agent

function KB-AGENT(*percept*) **returns** an *action*
  static: *KB*, a knowledge base // background information
    t, a counter, initially 0, indicating time

  TELL(*KB*, MAKE-PERCEPT-SENTENCE(*percept*, *t*))
  action ← ASK(*KB*, MAKE-ACTION-QUERY(*t*)) // extensive reasoning about ??
  TELL(*KB*, MAKE-ACTION-SENTENCE(*action*, *t*))
  $t \leftarrow t + 1$
  return *action*

The agent must be able to:
- Represent states, actions, etc.
- Incorporate new percepts
- Update internal representations of the world
- Deduce hidden properties of the world
- Deduce appropriate actions

Wumpus World PEAS description

Performance measure

gold +1000, death -1000

-1 per step, -10 for using the arrow

Environment

Squares adjacent to wumpus are smelly

Squares adjacent to pit are breezy

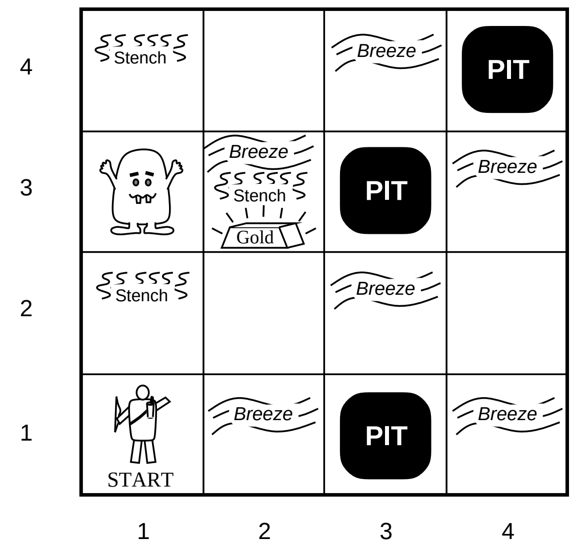
Glitter iff gold is in the same square

Shooting kills wumpus if you are facing it

Shooting uses up the only arrow

Grabbing picks up gold if in same square

Releasing drops the gold in same square



Random distribution of gold and W.
PIT with 20% prob.

Actuators Left turn, Right turn,
Forward, Grab, Release, Shoot

Sensors Breeze, Glitter, Smell , Bump, Scream

[Stench, Breeze, Glitter, Bump, Scream]

Wumpus world characterization

Observable??

Wumpus world characterization

Observable?? No—only local perception Partially observable

Deterministic??

Wumpus world characterization

Observable?? No—only local perception

Deterministic?? Yes—outcomes exactly specified

Episodic??

Wumpus world characterization

Observable?? No—only local perception

Deterministic?? Yes—outcomes exactly specified

Episodic?? No—sequential at the level of actions

Static??

Wumpus world characterization

Observable?? No—only local perception

Deterministic?? Yes—outcomes exactly specified

Episodic?? No—sequential at the level of actions

Static?? Yes—Wumpus and Pits do not move

Discrete??

Wumpus world characterization

Observable?? No—only local perception

Deterministic?? Yes—outcomes exactly specified

Episodic?? No—sequential at the level of actions

Static?? Yes—Wumpus and Pits do not move

Discrete?? Yes

Single-agent??

Wumpus world characterization

Observable?? No—only local perception

Deterministic?? Yes—outcomes exactly specified

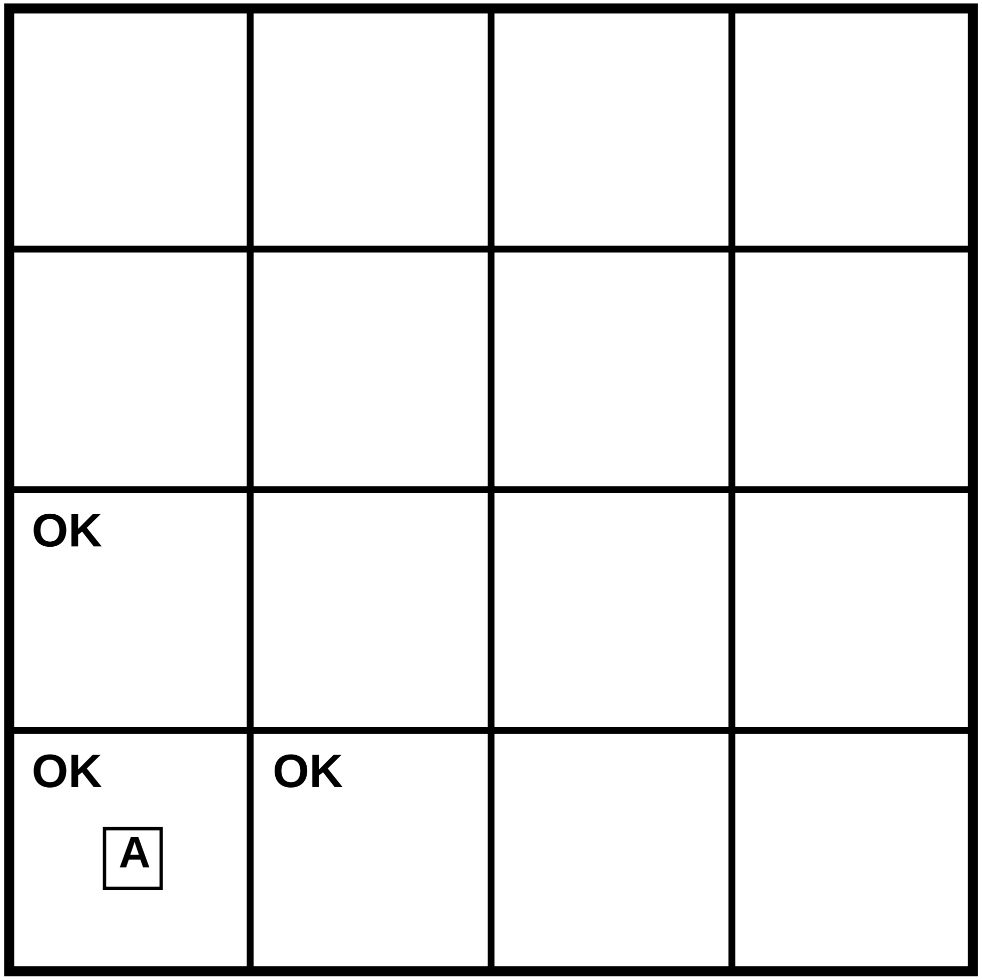
Episodic?? No—sequential at the level of actions

Static?? Yes—Wumpus and Pits do not move

Discrete?? Yes

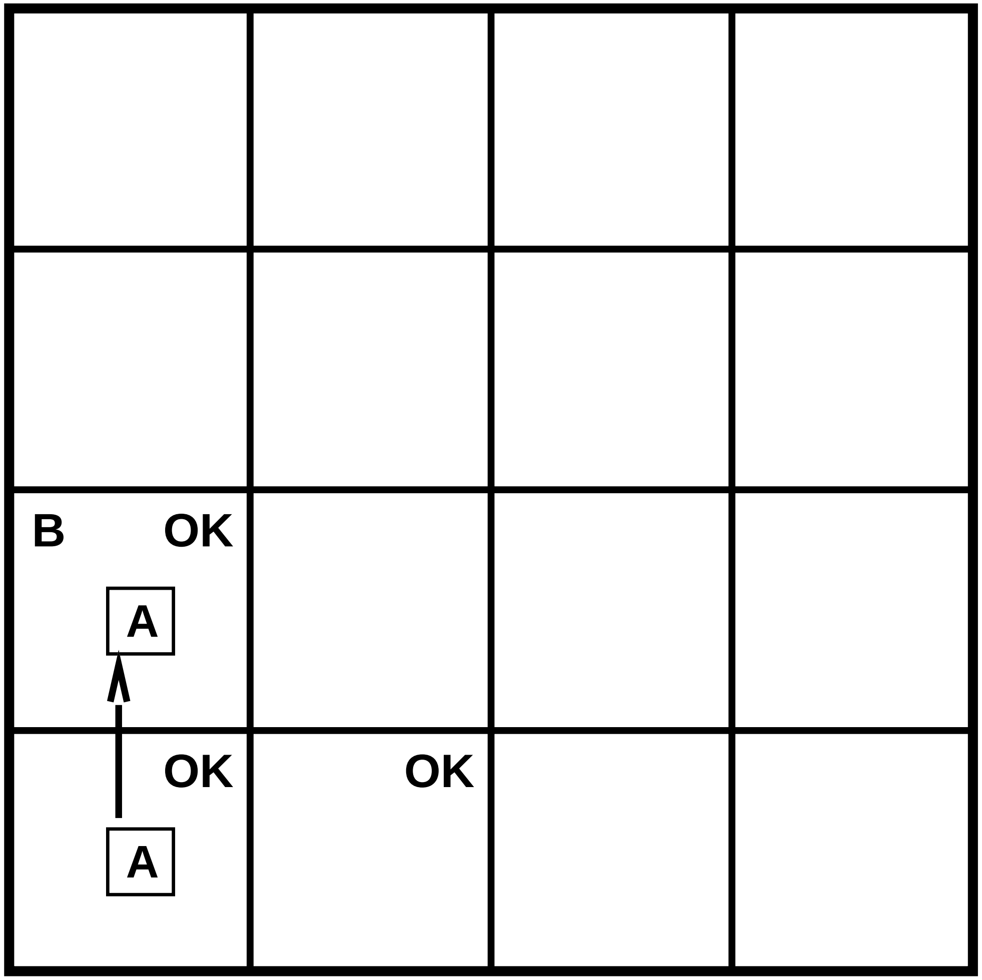
Single-agent?? Yes—Wumpus is essentially a natural feature

Exploring a wumpus world

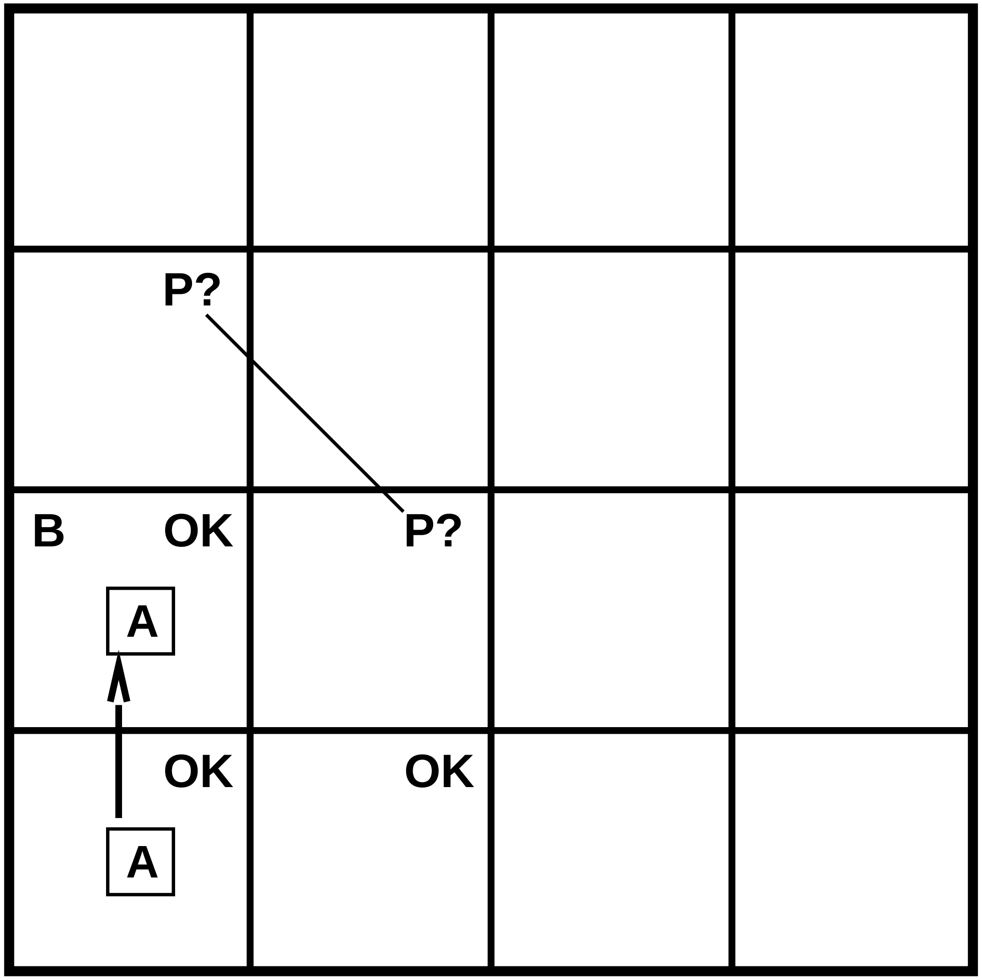


[None, None, None, None, None]

Exploring a wumpus world



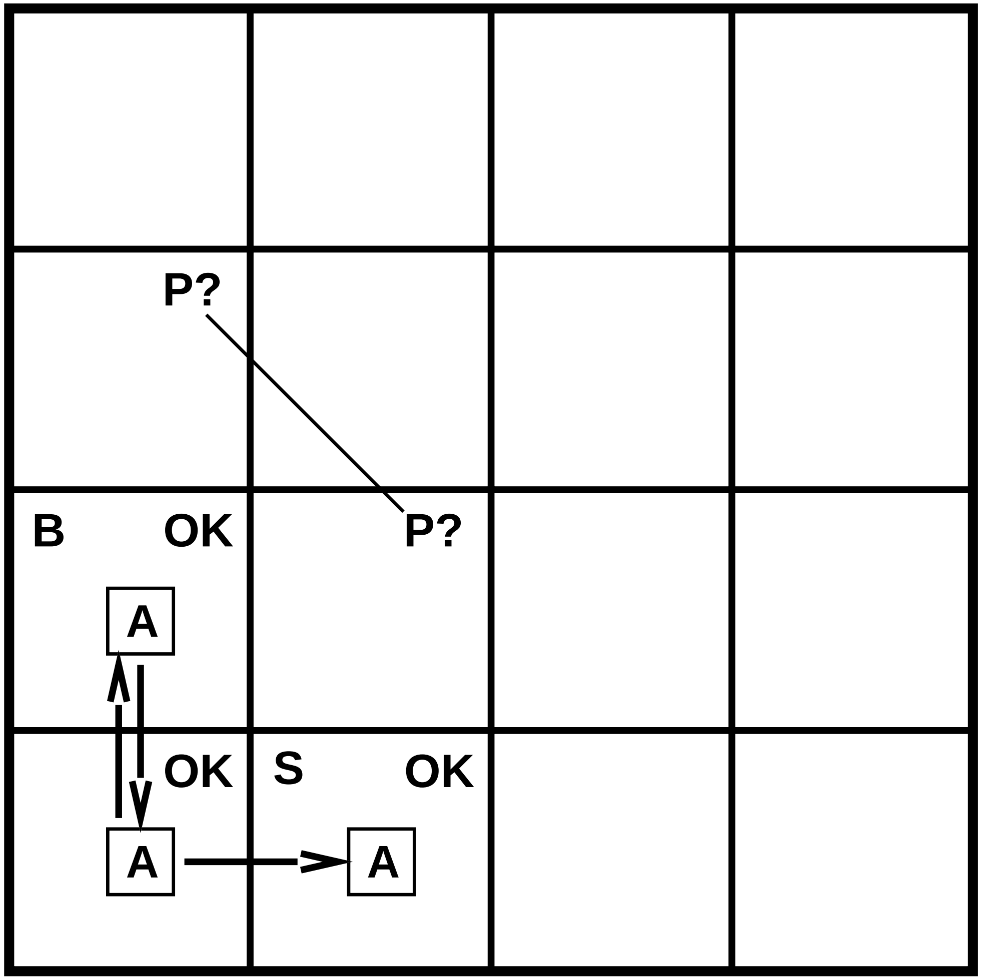
Exploring a wumpus world



[None, Breeze, None, None, None]

what to do?

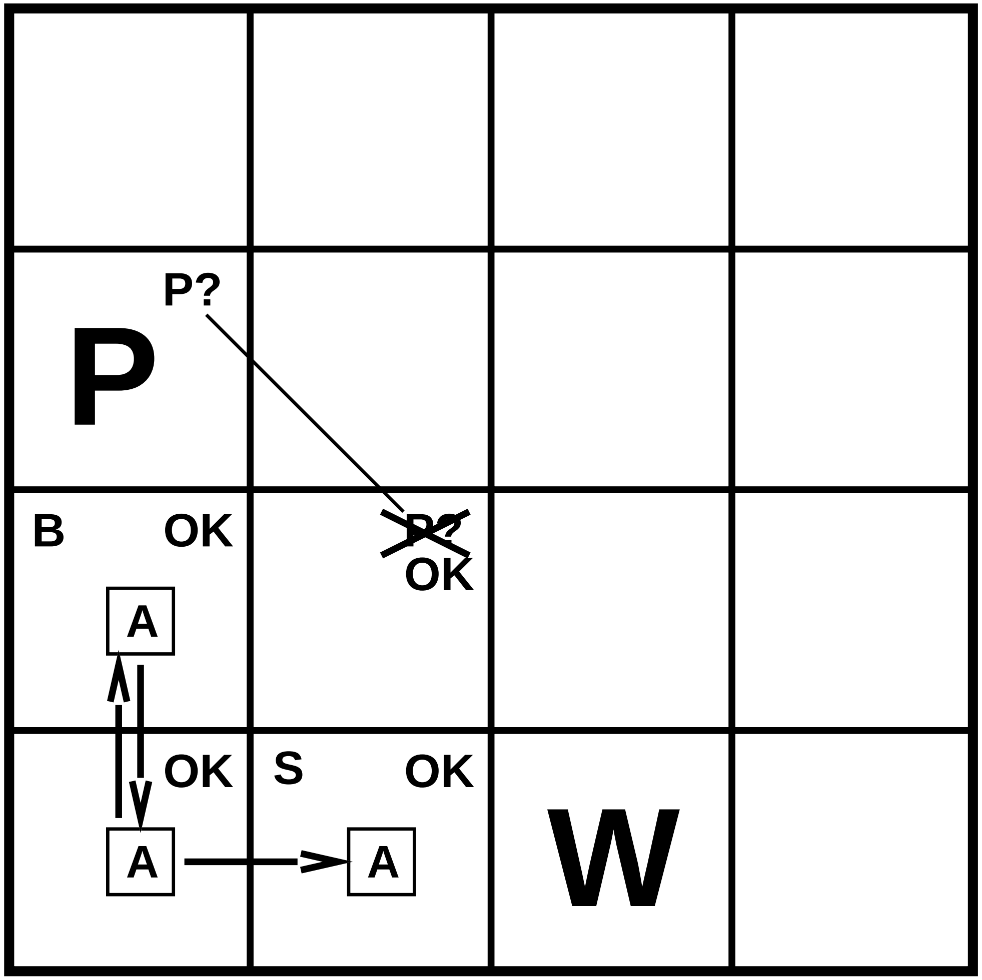
Exploring a wumpus world



[Stench, None, None, None, None]

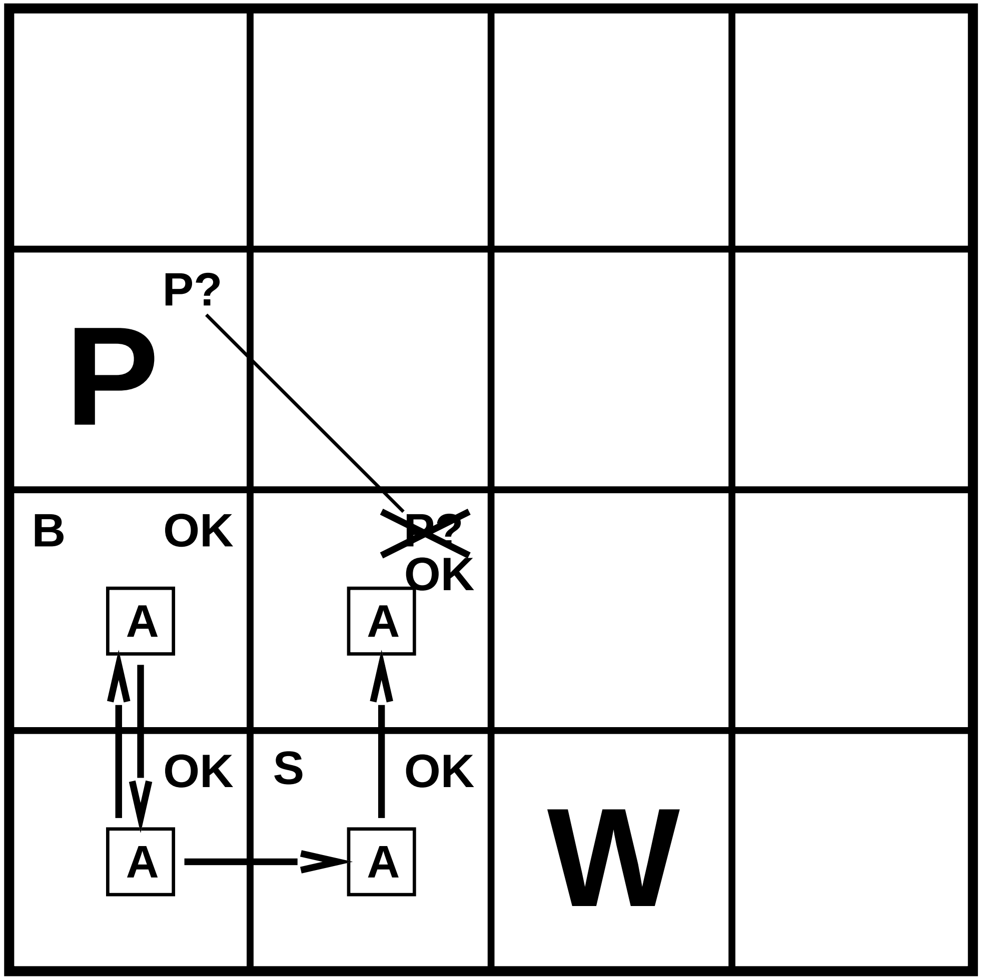
where is the wumpus?

Exploring a wumpus world

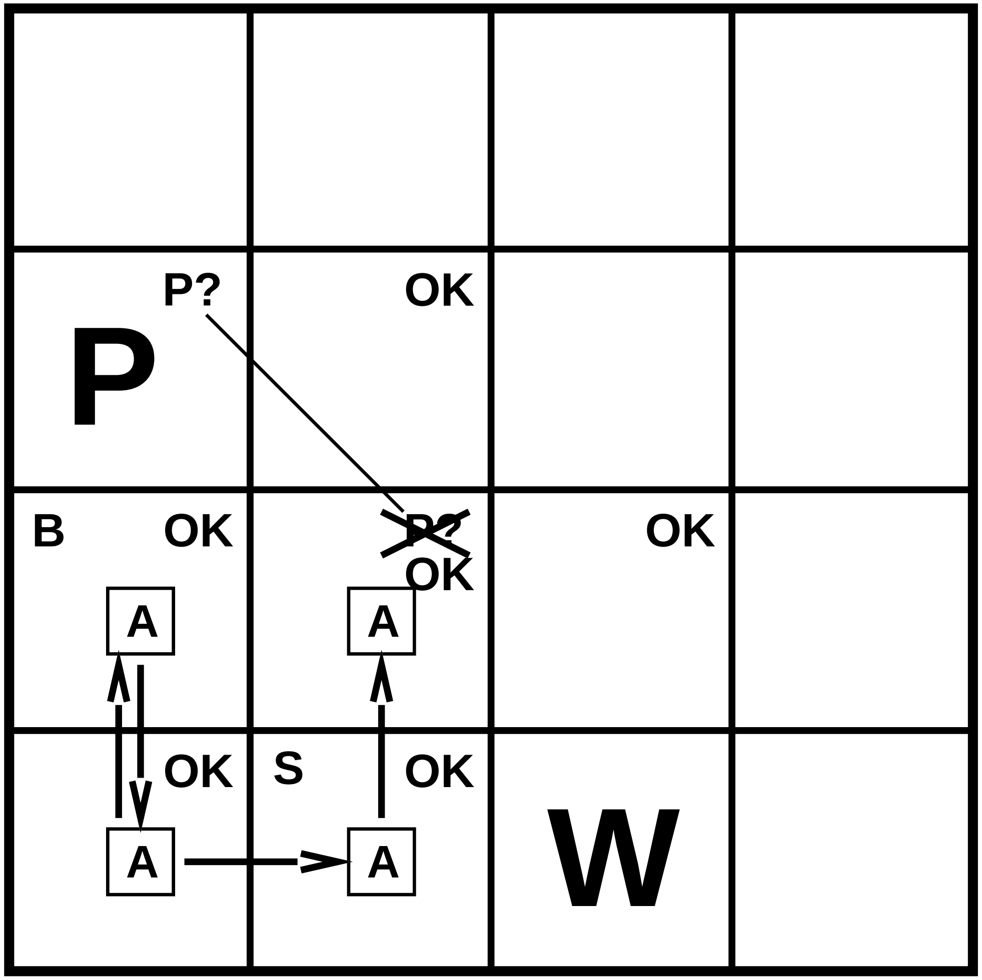


Was difficult: combined the knowledge gained at different times in different places
Difficult for animals.
this is what logical agents do

Exploring a wumpus world

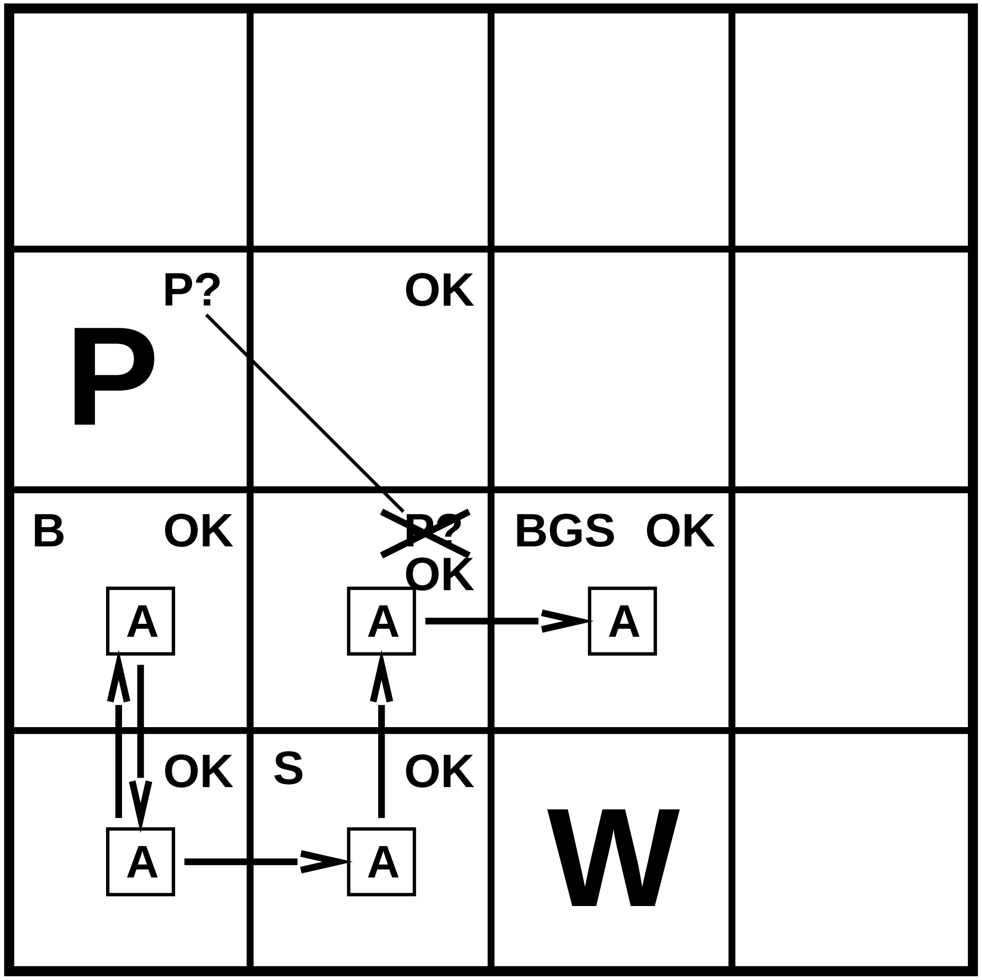


Exploring a wumpus world



[None, None, None, None, None]

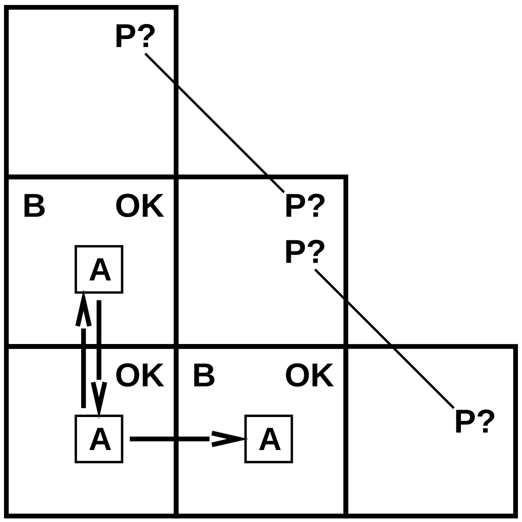
Exploring a wumpus world



[Stench, Breeze, Glitter, None, None]

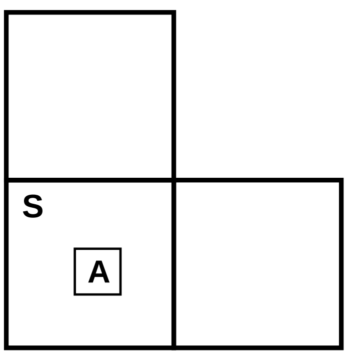
In each case, where the agent draws a conclusion from the available information, that conclusion is guaranteed to be correct if the available information is correct.

Other tight spots



Breeze in (1,2) and (2,1)
ctions

ormly distributed,
rob 0.86, vs. 0.31



Smell in (1,1)
$\Rightarrow$ cannot move
Can use a strategy of coercion:
shoot straight ahead
wumpus was there $\Rightarrow$ dead $\Rightarrow$ safe
wumpus wasn't there $\Rightarrow$ safe

Logic in general

Logics are formal languages for representing information
such that conclusions can be drawn

Syntax defines the sentences in the language

Semantics define the "meaning" of sentences;
i.e., define truth of a sentence in a world

E.g., the language of arithmetic

$x + 2 \geq y$ is a sentence; $x2 + y >$ is not a sentence

$x + 2 \geq y$ is true iff the number $x + 2$ is no less than the number y

$x + 2 \geq y$ is true in a world where $x = 7,\ y = 1$
$x + 2 \geq y$ is false in a world where $x = 0,\ y = 6$

to be precise, use "model" instead of "possible world"
--> mathematical abstraction

Entailment

Entailment means that one thing **follows from** another:

$$KB \models \alpha$$

Knowledge base KB entails sentence α
if and only if
α is true in all worlds where KB is true

E.g., the KB containing "the Giants won" and "the Reds won" entails "Either the Giants won or the Reds won"

E.g., $x + y = 4$ entails $4 = x + y$

Entailment is a relationship between sentences (i.e., **syntax**) that is based on **semantics**

Note: brains process **syntax** (of some sort)

Models

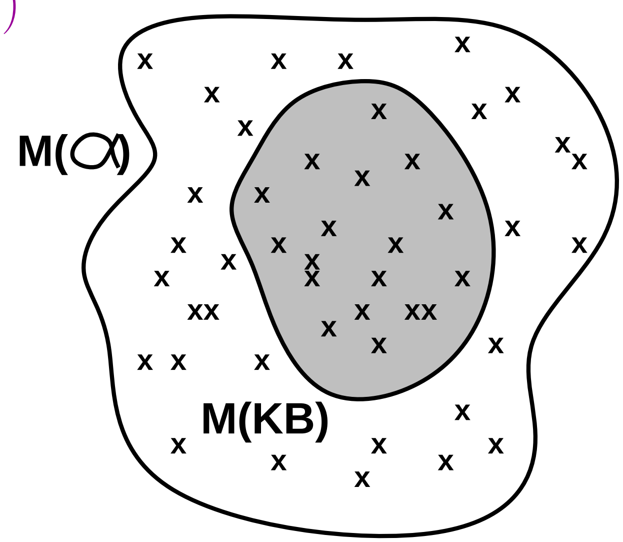
Logicians typically think in terms of models, which are formally structured worlds with respect to which truth can be evaluated

We say m is a model of a sentence α if α is true in m

$M(\alpha)$ is the set of all models of α

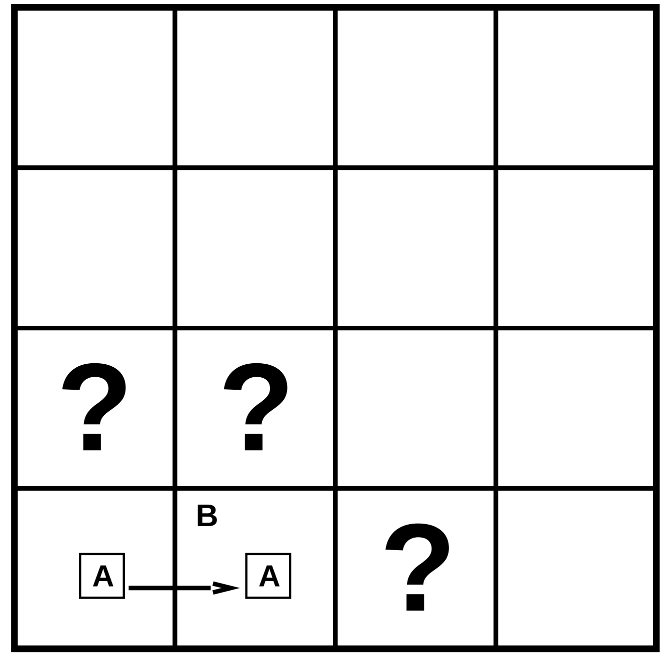
Then $KB \models \alpha$ if and only if $M(KB) \subseteq M(\alpha)$

E.g. $KB =$ Giants won and Reds won
$\alpha =$ Giants won



a |= b if and only if, in every model in which a is true, b is also true.

Entailment in the wumpus world

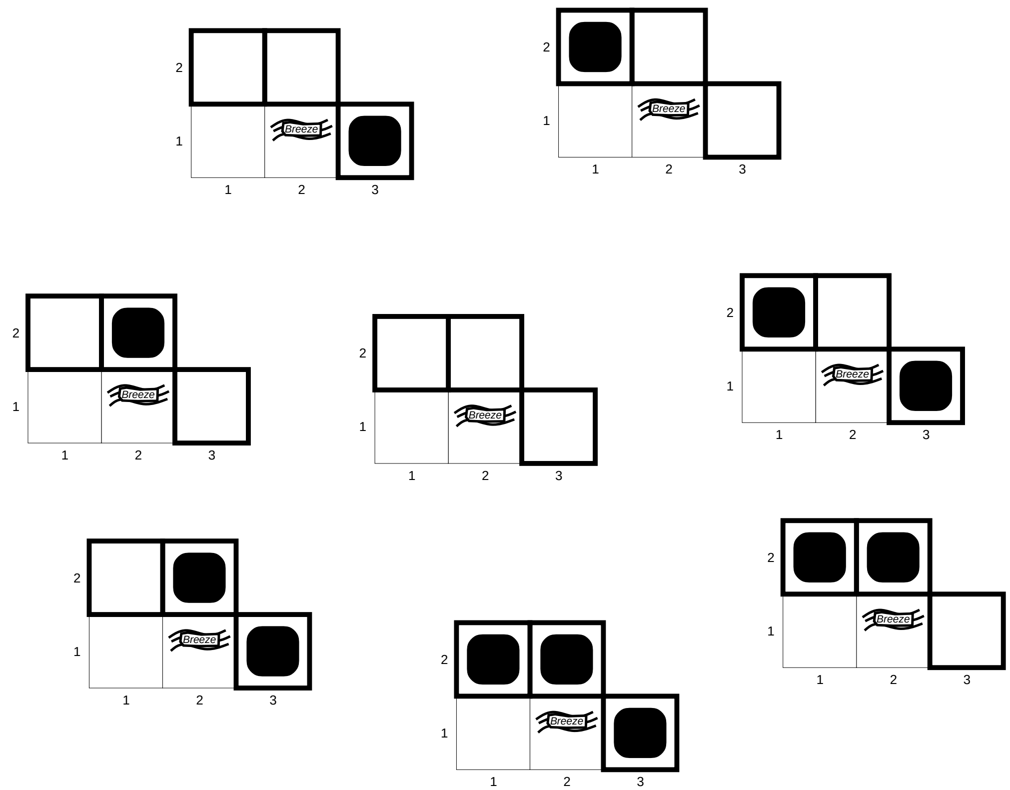


Situation after detecting nothing in [1,1], moving right, breeze in [2,1]

Consider possible models for ?s assuming only pits

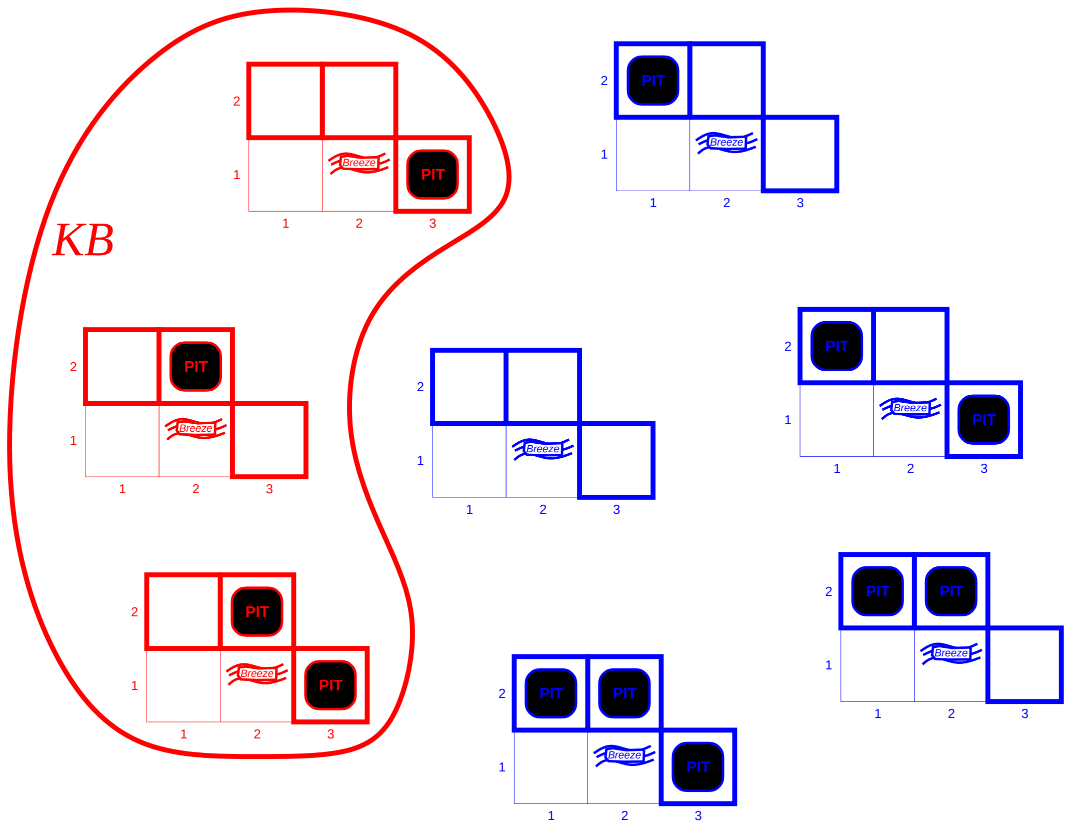
3 Boolean choices $\Rightarrow$ 8 possible models

Wumpus models



Q: Which models cover the
situation after detecting nothing in [1,1], moving right, breeze in [2,1]

Wumpus models

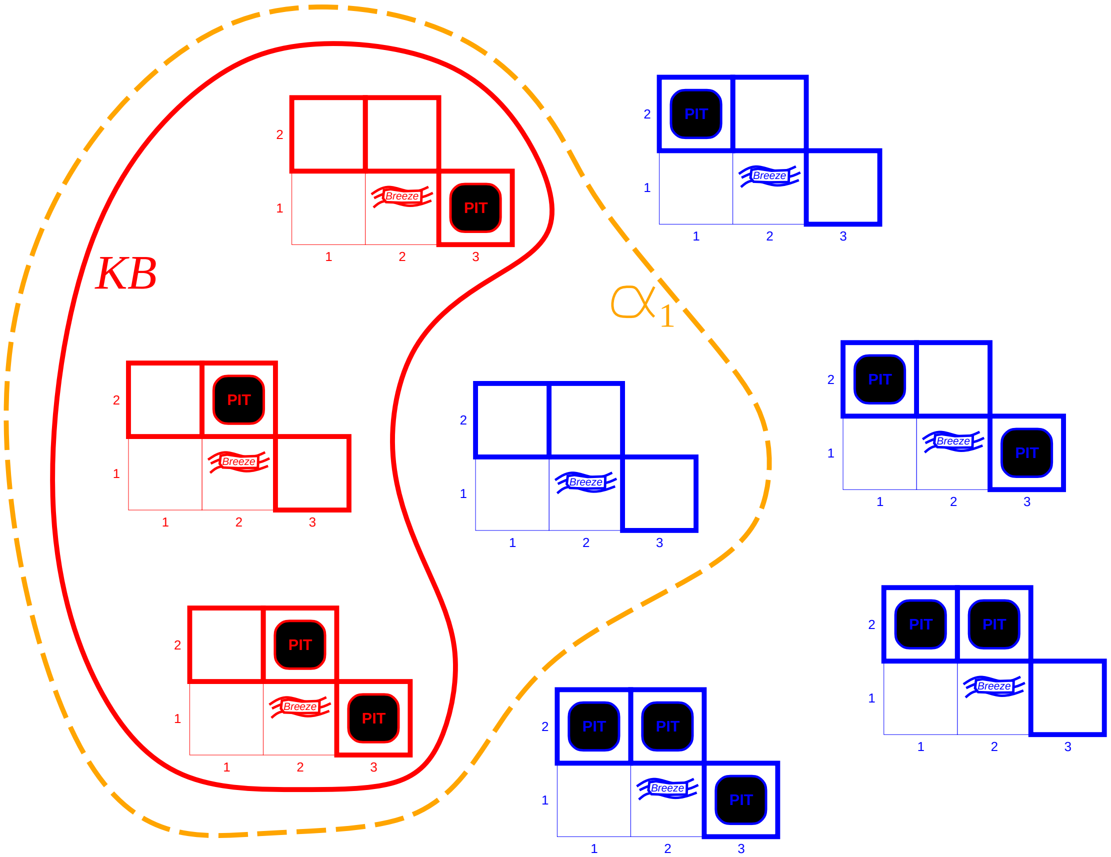


KB = wumpus-world rules + observations

There are three models in which KB is true.

Q: In which models, [1,2] is safe?

Wumpus models



There is no pit in [1,2].

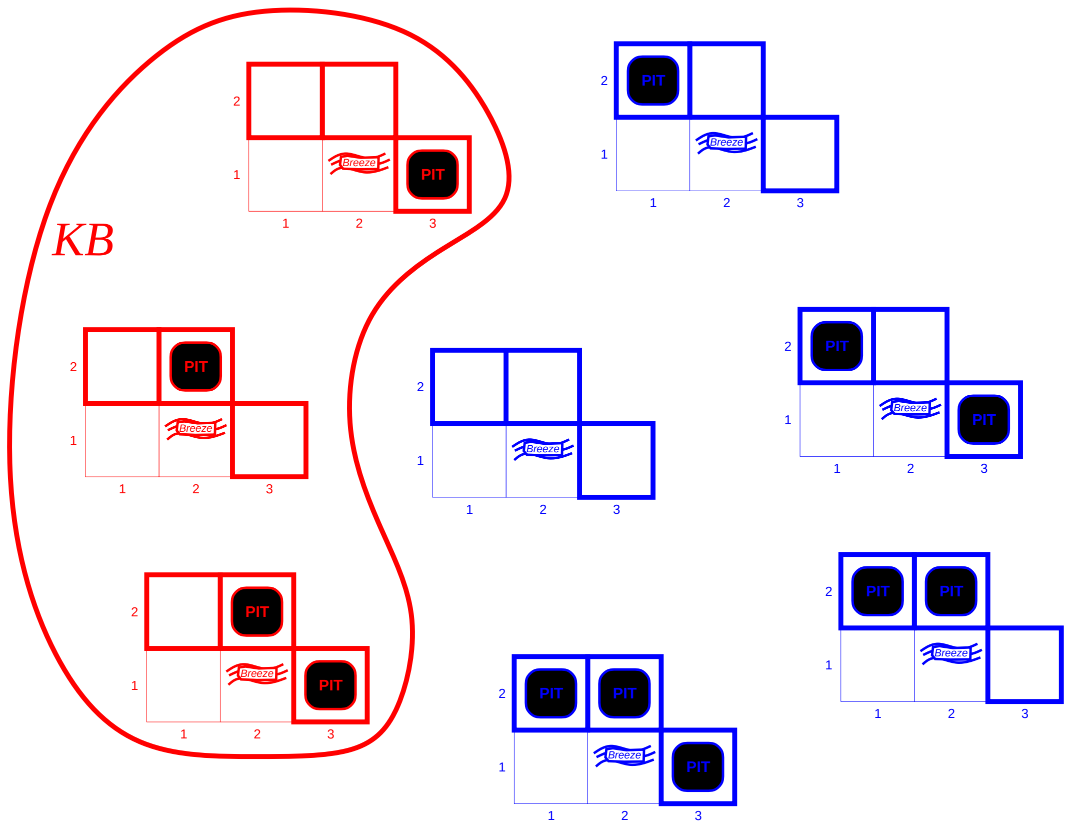
KB = wumpus-world rules + observations

α_1 = "[1,2] is safe", $KB \models \alpha_1$, proved by model checking

- enumerate all possibble models to check that alpha is true in all models in which KB is true.

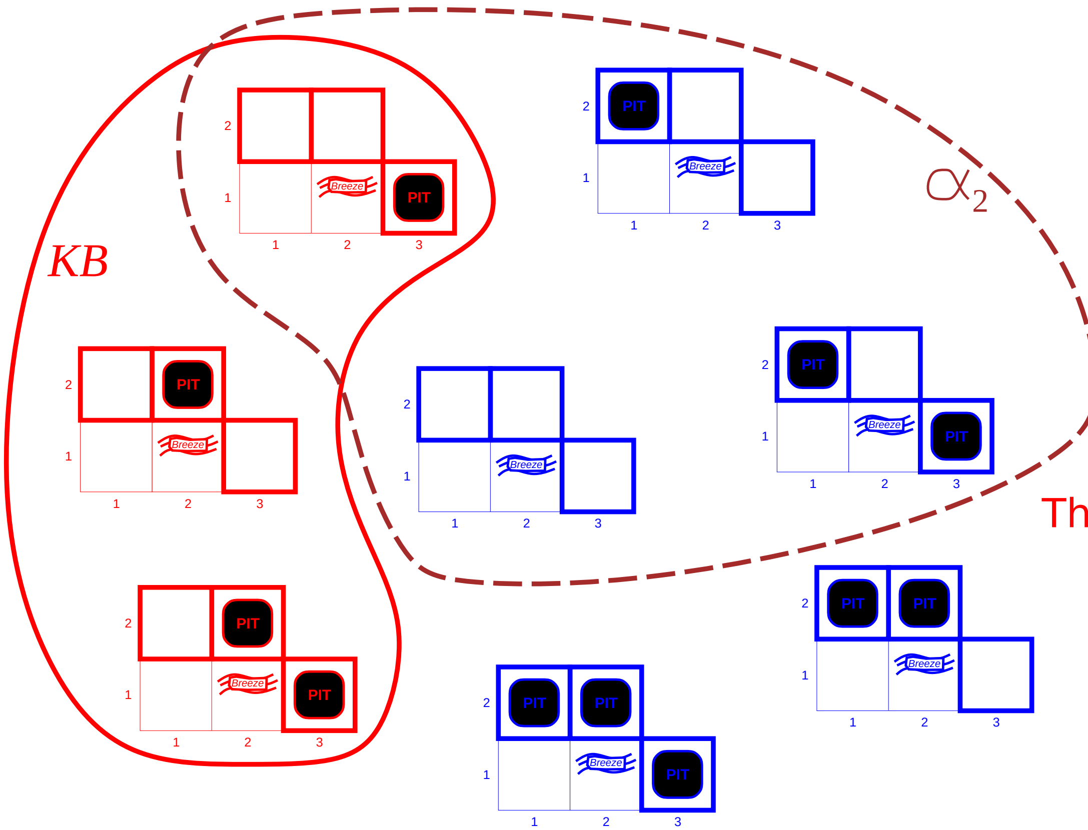
In every model KB is true, a1 is also true.

Wumpus models



KB = wumpus-world rules + observations

Wumpus models



There is no pit in [2,2].

KB = wumpus-world rules + observations

α_2 = "[2,2] is safe", $KB \not\models \alpha_2$

in some models in which KB is true, a2 is false.

Prev. inference algorithm: model checking

Deriving to conclusions: Inference

$KB \vdash_i \alpha$ = sentence α can be derived from KB by procedure i

Consequences of KB are a haystack; α is a needle.
Entailment = needle in haystack; inference = finding it

Soundness: i is sound if
whenever $KB \vdash_i \alpha$, it is also true that $KB \models \alpha$
i.e. an inference algorithm that derives only entailed sentences is called sound.

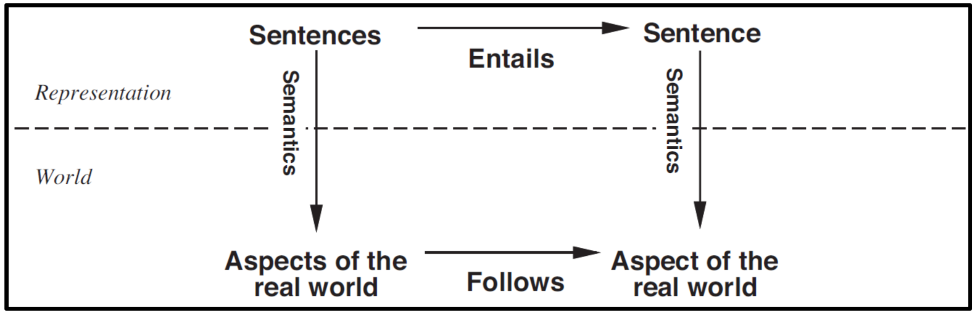
Completeness: i is complete if
whenever $KB \models \alpha$, it is also true that $KB \vdash_i \alpha$
i.e. an inference algorithm is complete if it can derive any sentence that is entailed.

Preview: we will define a logic (first-order logic) which is expressive enough to say almost anything of interest, and for which there exists a sound and complete inference procedure.

That is, the procedure will answer any question whose answer follows from what is known by the KB.

Reasoning process

- If KB is true in the real world, then any sentence α derived from KB by a sound inference procedure is also true in the real world.
  - Inference operates on syntax



- How do we know that KB is true in real world?
  - Syntax is in agent's head
  - **Symbol grounding**

Cognitive Development and Symbol Emergence in Humans and Robots

Posted by kikakuka on Oct 31, 2015 in Seminars | Comments Off on Cognitive Development and Symbol Emergence in Humans and Robots

NII Shonan Meeting:

@ Shonan Village Center, October 3-7, 2016

Organizers

- Tadahiro Taniguchi, Ritsumeikan University, Japan
- Emre Ugur, Bogazici University, Turkey
- George Konidaris, Duke University, US

Propositional logic: Syntax

Boolean logic (George Boole 1815-1864)

Propositional logic is the simplest logic—illustrates basic ideas

The proposition symbols P_1, P_2 etc are sentences

If S is a sentence, $\neg S$ is a sentence (negation)

If S_1 and S_2 are sentences, $S_1 \wedge S_2$ is a sentence (conjunction)

If S_1 and S_2 are sentences, $S_1 \vee S_2$ is a sentence (disjunction)

If S_1 and S_2 are sentences, $S_1 \Rightarrow S_2$ is a sentence (implication)

If S_1 and S_2 are sentences, $S_1 \Leftrightarrow S_2$ is a sentence (biconditional)

True and False are also proposition symbols

A literal is either an atomic sentence or a negative atomic sentence.

Propositional Logic : Syntax

$$
\begin{array}{rcl}
Sentence & \rightarrow & AtomicSentence \mid ComplexSentence \\
AtomicSentence & \rightarrow & True \mid False \mid P \mid Q \mid R \mid \ldots \\
ComplexSentence & \rightarrow & (\, Sentence \,) \mid [\, Sentence \,] \\
& \mid & \neg\, Sentence \\
& \mid & Sentence \land Sentence \\
& \mid & Sentence \lor Sentence \\
& \mid & Sentence \;\Rightarrow\; Sentence \\
& \mid & Sentence \;\Leftrightarrow\; Sentence \\
\\
\text{OPERATOR PRECEDENCE} & : & \neg, \land, \lor, \Rightarrow, \Leftrightarrow
\end{array}
$$

Figure 7.7 A BNF (Backus–Naur Form) grammar of sentences in propositional logic, along with operator precedences, from highest to lowest.

Propositional logic: Semantics

Each model specifies true/false for each proposition symbol

E.g.	$P_{1,2}$	$P_{2,2}$	$P_{3,1}$
.	*false*	*false*	*true*

(With these symbols, 8 possible models, can be enumerated automatically.)

Rules for evaluating truth with respect to a model m:

$\neg S$	is true iff	S	is false		
$S_1 \wedge S_2$	is true iff	S_1	is true **and**	S_2	is true
$S_1 \vee S_2$	is true iff	S_1	is true **or**	S_2	is true
$S_1 \Rightarrow S_2$	is true iff	S_1	is false **or**	S_2	is true
i.e.,	is false iff	S_1	is true **and**	S_2	is false
$S_1 \Leftrightarrow S_2$	is true iff	$S_1 \Rightarrow S_2$	is true **and**	$S_2 \Rightarrow S_1$	is true

Simple recursive process evaluates an arbitrary sentence, e.g.,
$\neg P_{1,2} \wedge (P_{2,2} \vee P_{3,1}) = true \wedge (false \vee true) = true \wedge true = true$

Truth tables for connectives

P	Q	$\neg P$	$P \wedge Q$	$P \vee Q$	$P \Rightarrow Q$	$P \Leftrightarrow Q$
false	*false*	*true*	*false*	*false*	*true*	*true*
false	*true*	*true*	*false*	*true*	*true*	*false*
true	*false*	*false*	*false*	*true*	*false*	*false*
true	*true*	*false*	*true*	*true*	*true*	*true*

If P is true, then I am claiming Q is true. Otherwise, I have no claim.

Wumpus world sentences

Let $P_{i,j}$ be true if there is a pit in $[i,j]$.
Let $B_{i,j}$ be true if there is a breeze in $[i,j]$.

$$\neg P_{1,1}$$
$$\neg B_{1,1}$$
$$B_{2,1}$$

"Pits cause breezes in adjacent squares"

Wumpus world sentences

Let $P_{i,j}$ be true if there is a pit in $[i, j]$.
Let $B_{i,j}$ be true if there is a breeze in $[i, j]$.

$\neg P_{1,1}$
$\neg B_{1,1}$ what is the difference?
$B_{2,1}$

"Pits cause breezes in adjacent squares"

$$B_{1,1} \Leftrightarrow (P_{1,2} \vee P_{2,1})$$
$$B_{2,1} \Leftrightarrow (P_{1,1} \vee P_{2,2} \vee P_{3,1})$$

"A square is breezy **if and only if** there is an adjacent pit"

How can we construct the knowledge base?

Wumpus world sentences

Let $P_{i,j}$ be true if there is a pit in $[i, j]$.
Let $B_{i,j}$ be true if there is a breeze in $[i, j]$.

R1: $\neg P_{1,1}$

R4: $\neg B_{1,1}$

R5: $B_{2,1}$

"Pits cause breezes in adjacent squares"

R2: $B_{1,1} \quad \Leftrightarrow \quad (P_{1,2} \lor P_{2,1})$

R3: $B_{2,1} \quad \Leftrightarrow \quad (P_{1,1} \lor P_{2,2} \lor P_{3,1})$

"A square is breezy **if and only if** there is an adjacent pit"

Recall that the aim of logical inference is to decide whether KB|=a for some sentence.

Truth tables for inference

$B_{1,1}$	$B_{2,1}$	$P_{1,1}$	$P_{1,2}$	$P_{2,1}$	$P_{2,2}$	$P_{3,1}$	R_1	R_2	R_3	R_4	R_5	KB
false	*false*	*false*	*false*	*false*	*false*	*false*	*true*	*true*	*true*	*true*	*false*	*false*
false	*false*	*false*	*false*	*false*	*false*	*true*	*true*	*true*	*false*	*true*	*false*	*false*
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮
false	*true*	*false*	*false*	*false*	*false*	*false*	*true*	*true*	*false*	*true*	*true*	*false*
false	*true*	*false*	*false*	*false*	*false*	*true*	*true*	*true*	*true*	*true*	*true*	*true*
false	*true*	*false*	*false*	*false*	*true*	*false*	*true*	*true*	*true*	*true*	*true*	*true*
false	*true*	*false*	*false*	*false*	*true*	*true*	*true*	*true*	*true*	*true*	*true*	*true*
false	*true*	*false*	*false*	*true*	*false*	*false*	*true*	*false*	*false*	*true*	*true*	*false*
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮
true	*true*	*true*	*true*	*true*	*true*	*true*	*false*	*true*	*true*	*false*	*true*	*false*

Enumerate rows (different assignments to symbols),
if KB is true in row, check that α is too

i.e. enumerate the models, and check that alpha is true in every model in which KB is true.

how many models are there in total?

where is the pit? -- need to reformulate as a model (or models).

Inference by Enumeration

function TT-ENTAILS?(KB, α) **returns** *true* or *false*
  inputs: KB, the knowledge base, a sentence in propositional logic
    α, the query, a sentence in propositional logic

  $symbols \leftarrow$ a list of the proposition symbols in KB and α
  return TT-CHECK-ALL($KB, \alpha, symbols, \{\ \}$)

---

function TT-CHECK-ALL($KB, \alpha, symbols, model$) **returns** *true* or *false*
  if EMPTY?($symbols$) **then**
    if PL-TRUE?($KB, model$) **then return** PL-TRUE?($\alpha, model$)
    else return *true* // *when KB is false, always return true*
  else do
    $P \leftarrow$ FIRST($symbols$)
    $rest \leftarrow$ REST($symbols$)
    return (TT-CHECK-ALL($KB, \alpha, rest, model \cup \{P = true\}$)
      and
      TT-CHECK-ALL($KB, \alpha, rest, model \cup \{P = false\}$))

Figure 7.10 A truth-table enumeration algorithm for deciding propositional entailment. (TT stands for truth table.) PL-TRUE? returns *true* if a sentence holds within a model. The variable *model* represents a partial model—an assignment to some of the symbols. The key-word "**and**" is used here as a logical operation on its two arguments, returning *true* or *false*.

Inference by enumeration

Depth-first enumeration of all models is sound and complete

```
function TT-Entails?(KB, α) returns true or false
   inputs: KB, the knowledge base, a sentence in propositional logic
           α, the query, a sentence in propositional logic

   symbols ← a list of the proposition symbols in KB and α
   return TT-Check-All(KB, α, symbols, [ ])
```

```
function TT-Check-All(KB, α, symbols, model) returns true or false
   if Empty?(symbols) then
      if PL-True?(KB, model) then return PL-True?(α, model)
      else return true
   else do
      P ← First(symbols); rest ← Rest(symbols)
      return TT-Check-All(KB, α, rest, Extend(P, true, model)) and
                 TT-Check-All(KB, α, rest, Extend(P, false, model))
```

$O(2^n)$ for n symbols;

Logical equivalence

Two sentences are logically equivalent iff true in same models:
$\alpha \equiv \beta$ if and only if $\alpha \models \beta$ and $\beta \models \alpha$

$$
\begin{array}{rcll}
(\alpha \land \beta) & \equiv & (\beta \land \alpha) & \text{commutativity of } \land \\
(\alpha \lor \beta) & \equiv & (\beta \lor \alpha) & \text{commutativity of } \lor \\
((\alpha \land \beta) \land \gamma) & \equiv & (\alpha \land (\beta \land \gamma)) & \text{associativity of } \land \\
((\alpha \lor \beta) \lor \gamma) & \equiv & (\alpha \lor (\beta \lor \gamma)) & \text{associativity of } \lor \\
\neg(\neg\alpha) & \equiv & \alpha & \text{double-negation elimination} \\
(\alpha \Rightarrow \beta) & \equiv & (\neg\beta \Rightarrow \neg\alpha) & \text{contraposition} \\
(\alpha \Rightarrow \beta) & \equiv & (\neg\alpha \lor \beta) & \text{implication elimination} \\
(\alpha \Leftrightarrow \beta) & \equiv & ((\alpha \Rightarrow \beta) \land (\beta \Rightarrow \alpha)) & \text{biconditional elimination} \\
\neg(\alpha \land \beta) & \equiv & (\neg\alpha \lor \neg\beta) & \text{De Morgan} \\
\neg(\alpha \lor \beta) & \equiv & (\neg\alpha \land \neg\beta) & \text{De Morgan} \\
(\alpha \land (\beta \lor \gamma)) & \equiv & ((\alpha \land \beta) \lor (\alpha \land \gamma)) & \text{distributivity of } \land \text{ over } \lor \\
(\alpha \lor (\beta \land \gamma)) & \equiv & ((\alpha \lor \beta) \land (\alpha \lor \gamma)) & \text{distributivity of } \lor \text{ over } \land
\end{array}
$$

Validity and satisfiability

A sentence is valid if it is true in **all** models,

e.g., $True$,

Validity is connected to inference via the Deduction Theorem:

$KB \models \alpha$ if and only if $(KB \; \Rightarrow \; \alpha)$ is valid

A sentence is satisfiable if it is true in **some** model

e.g.,

A sentence is unsatisfiable if it is true in **no** models

e.g.,

Satisfiability is connected to inference via the following:

$KB \models \alpha$ if and only if $(KB \wedge \neg\alpha)$ is unsatisfiable

i.e., prove α by *reductio ad absurdum*

Validity and satisfiability

A sentence is valid if it is true in **all** models,

e.g., $True$, $A \vee \neg A$, $A \Rightarrow A$, $(A \wedge (A \Rightarrow B)) \Rightarrow B$

Validity is connected to inference via the Deduction Theorem:

$KB \models \alpha$ if and only if $(KB \Rightarrow \alpha)$ is valid

A sentence is satisfiable if it is true in **some** model

e.g., $A \vee B$, C

A sentence is unsatisfiable if it is true in **no** models

e.g., $A \wedge \neg A$

Satisfiability is connected to inference via the following:

$KB \models \alpha$ if and only if $(KB \wedge \neg\alpha)$ is unsatisfiable

i.e., prove α by *reductio ad absurdum*

Proof methods

Proof methods divide into (roughly) two kinds:

Application of inference rules

– Legitimate (sound) generation of new sentences from old
– Proof $=$ a sequence of inference rule applications
  Can use inference rules as operators in a standard search alg.
– Typically require translation of sentences into a normal form

Model checking

truth table enumeration (always exponential in n)
improved backtracking, e.g., Davis–Putnam–Logemann–Loveland
heuristic search in model space (sound but incomplete)
  e.g., min-conflicts-like hill-climbing algorithms

Proof with Inference rules

- Modus Ponens: $\dfrac{\alpha \Rightarrow \beta, \quad \alpha}{\beta}$
- And-Elimination: $\dfrac{\alpha \wedge \beta}{\alpha}$

$$
\begin{array}{rcll}
(\alpha \wedge \beta) & \equiv & (\beta \wedge \alpha) & \text{commutativity of } \wedge \\
(\alpha \vee \beta) & \equiv & (\beta \vee \alpha) & \text{commutativity of } \vee \\
((\alpha \wedge \beta) \wedge \gamma) & \equiv & (\alpha \wedge (\beta \wedge \gamma)) & \text{associativity of } \wedge \\
((\alpha \vee \beta) \vee \gamma) & \equiv & (\alpha \vee (\beta \vee \gamma)) & \text{associativity of } \vee \\
\neg(\neg\alpha) & \equiv & \alpha & \text{double-negation elimination} \\
(\alpha \Rightarrow \beta) & \equiv & (\neg\beta \Rightarrow \neg\alpha) & \text{contraposition} \\
(\alpha \Rightarrow \beta) & \equiv & (\neg\alpha \vee \beta) & \text{implication elimination} \\
(\alpha \Leftrightarrow \beta) & \equiv & ((\alpha \Rightarrow \beta) \wedge (\beta \Rightarrow \alpha)) & \text{biconditional eliminatio} \\
\neg(\alpha \wedge \beta) & \equiv & (\neg\alpha \vee \neg\beta) & \text{De Morgan} \\
\neg(\alpha \vee \beta) & \equiv & (\neg\alpha \wedge \neg\beta) & \text{De Morgan} \\
(\alpha \wedge (\beta \vee \gamma)) & \equiv & ((\alpha \wedge \beta) \vee (\alpha \wedge \gamma)) & \text{distributivity of } \wedge \text{ over } \vee \\
(\alpha \vee (\beta \wedge \gamma)) & \equiv & ((\alpha \vee \beta) \wedge (\alpha \vee \gamma)) & \text{distributivity of } \vee \text{ over } \wedge
\end{array}
$$

Proof with Inference rules – Quiz

$$R_1: \neg P_{1,1}$$
$$R_2: B_{1,1} \Leftrightarrow (P_{1,2} \vee P_{2,1}) .$$
$$R_3: B_{2,1} \Leftrightarrow (P_{1,1} \vee P_{2,2} \vee P_{3,1})$$
$$R_4: \neg B_{1,1} .$$
$$R_5: B_{2,1} .$$

$P_{2,1}$ Is there a pit at (2,1)?

- Bi-conditional elimination to R_2
- And-elimination
- Contrapositives
- Modus ponens with R_4
- De morgan's rule

$$\frac{\alpha \Rightarrow \beta, \quad \alpha}{\beta}$$

$$\frac{\alpha \wedge \beta}{\alpha}$$

$$\begin{array}{rcll}
(\alpha \wedge \beta) & \equiv & (\beta \wedge \alpha) & \text{commutativity of } \wedge \\
(\alpha \vee \beta) & \equiv & (\beta \vee \alpha) & \text{commutativity of } \vee \\
((\alpha \wedge \beta) \wedge \gamma) & \equiv & (\alpha \wedge (\beta \wedge \gamma)) & \text{associativity of } \wedge \\
((\alpha \vee \beta) \vee \gamma) & \equiv & (\alpha \vee (\beta \vee \gamma)) & \text{associativity of } \vee \\
\neg(\neg\alpha) & \equiv & \alpha & \text{double-negation elimination} \\
(\alpha \Rightarrow \beta) & \equiv & (\neg\beta \Rightarrow \neg\alpha) & \text{contraposition} \\
(\alpha \Rightarrow \beta) & \equiv & (\neg\alpha \vee \beta) & \text{implication elimination} \\
(\alpha \Leftrightarrow \beta) & \equiv & ((\alpha \Rightarrow \beta) \wedge (\beta \Rightarrow \alpha)) & \text{biconditional elimination} \\
\neg(\alpha \wedge \beta) & \equiv & (\neg\alpha \vee \neg\beta) & \text{De Morgan} \\
\neg(\alpha \vee \beta) & \equiv & (\neg\alpha \wedge \neg\beta) & \text{De Morgan} \\
(\alpha \wedge (\beta \vee \gamma)) & \equiv & ((\alpha \wedge \beta) \vee (\alpha \wedge \gamma)) & \text{distributivity of } \wedge \text{ over } \vee \\
(\alpha \vee (\beta \wedge \gamma)) & \equiv & ((\alpha \vee \beta) \wedge (\alpha \vee \gamma)) & \text{distributivity of } \vee \text{ over } \wedge
\end{array}$$

Proof with inference rules

- Does it look like a search problem?
  -
- Does it look like more efficient than enumerating models?
  -
- Is it affected by additional sentences?
  - If $KB \models \alpha$ then $KB \wedge \beta \models \alpha$?

A single inference rule that yields a complete inference algoritm when coupled with any complete search algorithm.

Resolution

Conjunctive Normal Form (CNF—universal)

$$\textbf{conjunction} \text{ of } \underbrace{\textbf{disjunctions} \text{ of } \textbf{literals}}_{\textbf{clauses}}$$

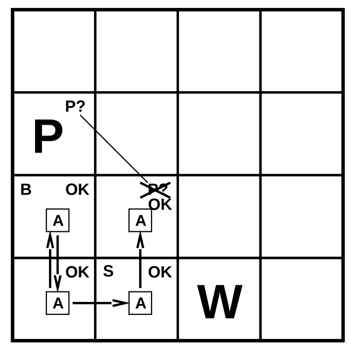
E.g., $(A \vee \neg B) \wedge (B \vee \neg C \vee \neg D)$

Resolution inference rule (for CNF): complete for propositional logic

$$\frac{\ell_1 \vee \cdots \vee \ell_k, \qquad m_1 \vee \cdots \vee m_n}{\ell_1 \vee \cdots \vee \ell_{i-1} \vee \ell_{i+1} \vee \cdots \vee \ell_k \vee m_1 \vee \cdots \vee m_{j-1} \vee m_{j+1} \vee \cdots \vee m_n}$$

where ℓ_i and m_j are complementary literals. E.g.,

$$\frac{P_{1,3} \vee P_{2,2}, \qquad \neg P_{2,2}}{P_{1,3}}$$



Resolution is sound and complete for propositional logic

Conversion to CNF

$B_{1,1} \Leftrightarrow (P_{1,2} \lor P_{2,1})$

1. Eliminate $\Leftrightarrow$, replacing $\alpha \Leftrightarrow \beta$ with $(\alpha \Rightarrow \beta) \land (\beta \Rightarrow \alpha)$.

   $(B_{1,1} \Rightarrow (P_{1,2} \lor P_{2,1})) \land ((P_{1,2} \lor P_{2,1}) \Rightarrow B_{1,1})$

2. Eliminate $\Rightarrow$, replacing $\alpha \Rightarrow \beta$ with $\neg\alpha \lor \beta$.

   $(\neg B_{1,1} \lor P_{1,2} \lor P_{2,1}) \land (\neg(P_{1,2} \lor P_{2,1}) \lor B_{1,1})$

3. Move $\neg$ inwards using de Morgan's rules and double-negation:

   $(\neg B_{1,1} \lor P_{1,2} \lor P_{2,1}) \land ((\neg P_{1,2} \land \neg P_{2,1}) \lor B_{1,1})$

4. Apply distributivity law ($\lor$ over $\land$) and flatten:

   $(\neg B_{1,1} \lor P_{1,2} \lor P_{2,1}) \land (\neg P_{1,2} \lor B_{1,1}) \land (\neg P_{2,1} \lor B_{1,1})$

Resolution algorithm

Proof by contradiction, i.e., show $KB \wedge \neg\alpha$ unsatisfiable

function PL-RESOLUTION(KB, α) **returns** *true* or *false*
  inputs: KB, the knowledge base, a sentence in propositional logic
    α, the query, a sentence in propositional logic

  $clauses \leftarrow$ the set of clauses in the CNF representation of $KB \wedge \neg\alpha$
  $new \leftarrow \{\}$
  loop do
    for each C_i, C_j **in** $clauses$ **do**
      $resolvents \leftarrow$ PL-RESOLVE(C_i, C_j)
      if $resolvents$ contains the empty clause **then return** *true*
      $new \leftarrow new \cup resolvents$
    if $new \subseteq clauses$ **then return** *false*
    $clauses \leftarrow clauses \cup new$

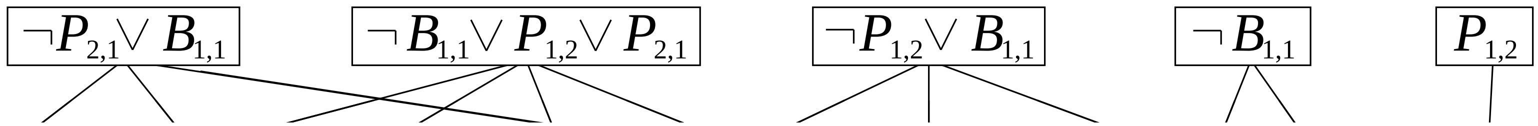
Each pair that contains complementary literals is resolved to produce a new clause
The new clause is added to the set if it is not already present. Continue until
- either there are no new clauses that can be added: KB does not entail alpha
- or two clauses resolve to yield the empty clause: KB entails alpha

Resolution example

$KB = (B_{1,1} \;\Leftrightarrow\; (P_{1,2} \vee P_{2,1})) \wedge \neg B_{1,1}$ $\alpha = \neg P_{1,2}$

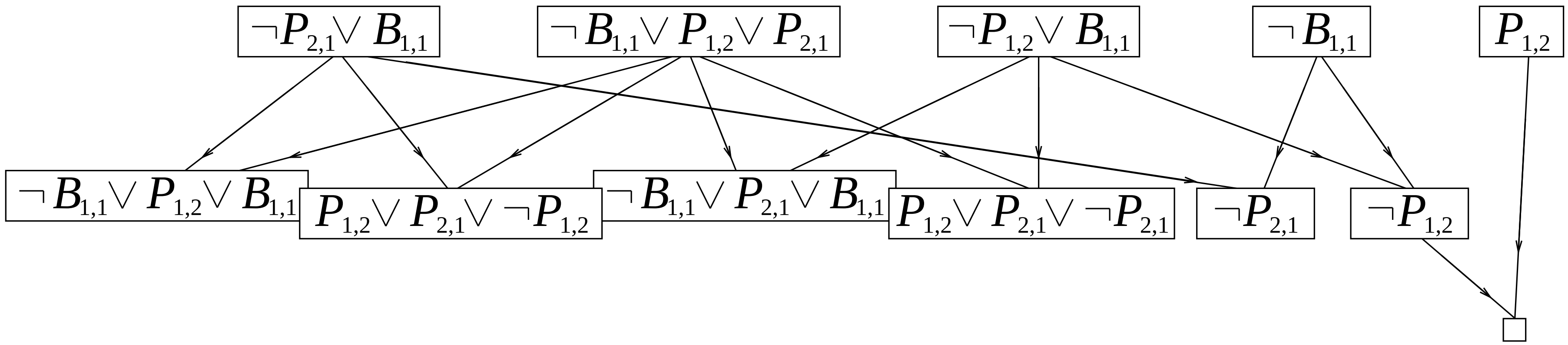
Resolution example

$KB = (B_{1,1} \;\Leftrightarrow\; (P_{1,2} \lor P_{2,1})) \land \neg B_{1,1}\;\; \alpha = \neg P_{1,2}$



Resolution example

$KB = (B_{1,1} \Leftrightarrow (P_{1,2} \vee P_{2,1})) \wedge \neg B_{1,1}$ $\alpha = \neg P_{1,2}$



Forward and Backward Chaining

- In many practical situations, the full power of resolution is not needed. Real-world knowledge bases often contain only clauses of a restricted kind, called **Horn clauses.**
  - Disjunction of literals with at most one positive literal

Forward and backward chaining

Horn Form (restricted)

KB = **conjunction** of **Horn clauses**

Horn clause =

- $\diamondsuit$ proposition symbol; or
- $\diamondsuit$ (conjunction of symbols) $\Rightarrow$ symbol

E.g., $C \wedge (B \Rightarrow A) \wedge (C \wedge D \Rightarrow B)$

Modus Ponens (for Horn Form): complete for Horn KBs

$$\frac{\alpha_1, \ldots, \alpha_n, \qquad \alpha_1 \wedge \cdots \wedge \alpha_n \Rightarrow \beta}{\beta}$$

Can be used with forward chaining or backward chaining.
These algorithms are very natural and run in **linear** time

Forward chaining

Idea: fire any rule whose premises are satisfied in the KB,
add its conclusion to the KB, until query is found

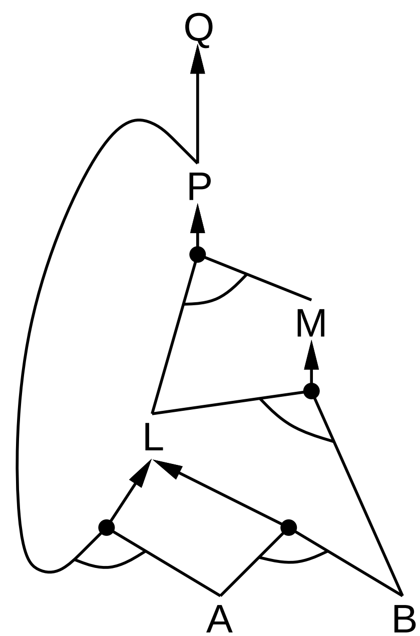
$$
\begin{aligned}
&P \Rightarrow Q \\
&L \wedge M \Rightarrow P \\
&B \wedge L \Rightarrow M \\
&A \wedge P \Rightarrow L \\
&A \wedge B \Rightarrow L \\
&A \\
&B
\end{aligned}
$$

Easy to see with AND-OR graphs

Forward chaining

Idea: fire any rule whose premises are satisfied in the KB,
add its conclusion to the KB, until query is found

$$P \Rightarrow Q$$
$$L \wedge M \Rightarrow P$$
$$B \wedge L \Rightarrow M$$
$$A \wedge P \Rightarrow L$$
$$A \wedge B \Rightarrow L$$
$$A$$
$$B$$

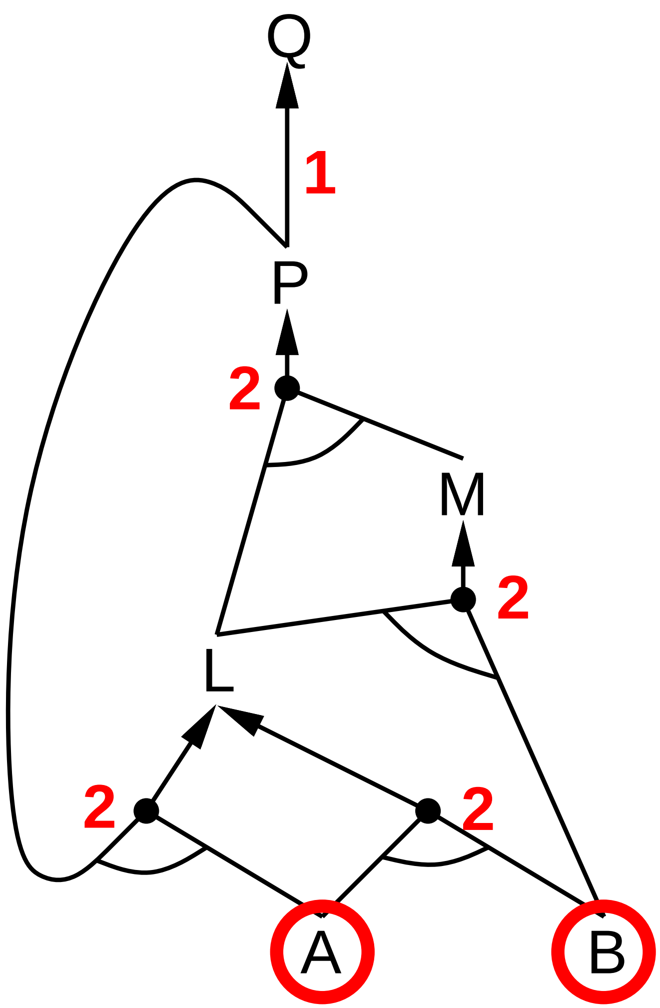


Forward chaining algorithm

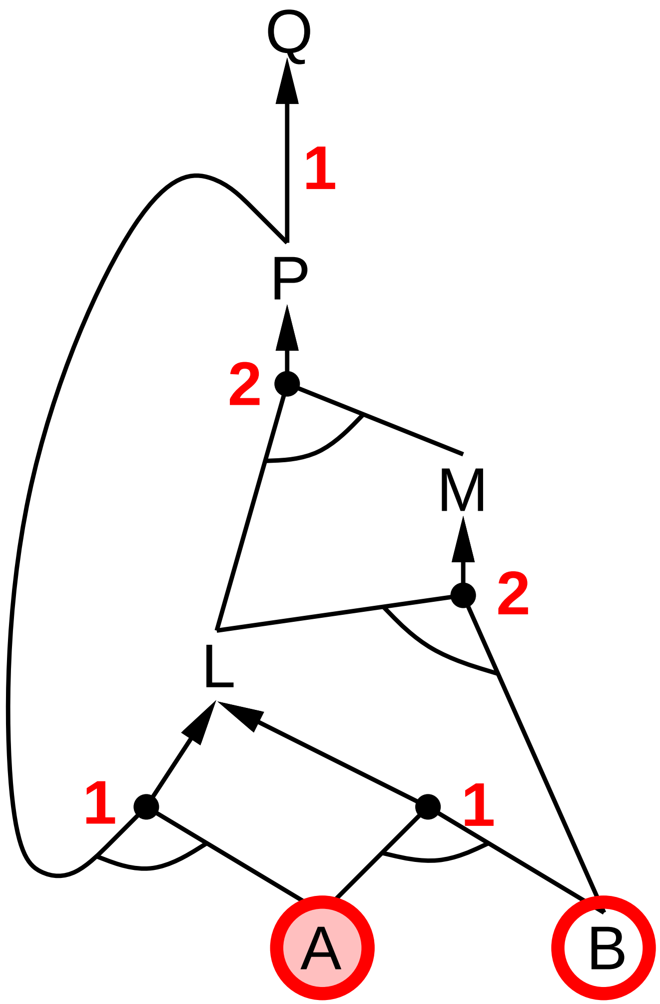
```
function PL-FC-ENTAILS?(KB, q) returns true or false
   inputs: KB, the knowledge base, a set of propositional Horn clauses
           q, the query, a proposition symbol
   local variables: count, a table, indexed by clause, initially the number of premises
                    inferred, a table, indexed by symbol, each entry initially false
                    agenda, a list of symbols, initially the symbols known in KB

   while agenda is not empty do
       p ← POP(agenda)
       unless inferred[p] do
           inferred[p] ← true
           for each Horn clause c in whose premise p appears do
               decrement count[c]
               if count[c] = 0 then do
                   if HEAD[c] = q then return true
                   PUSH(HEAD[c], agenda)
   return false
```

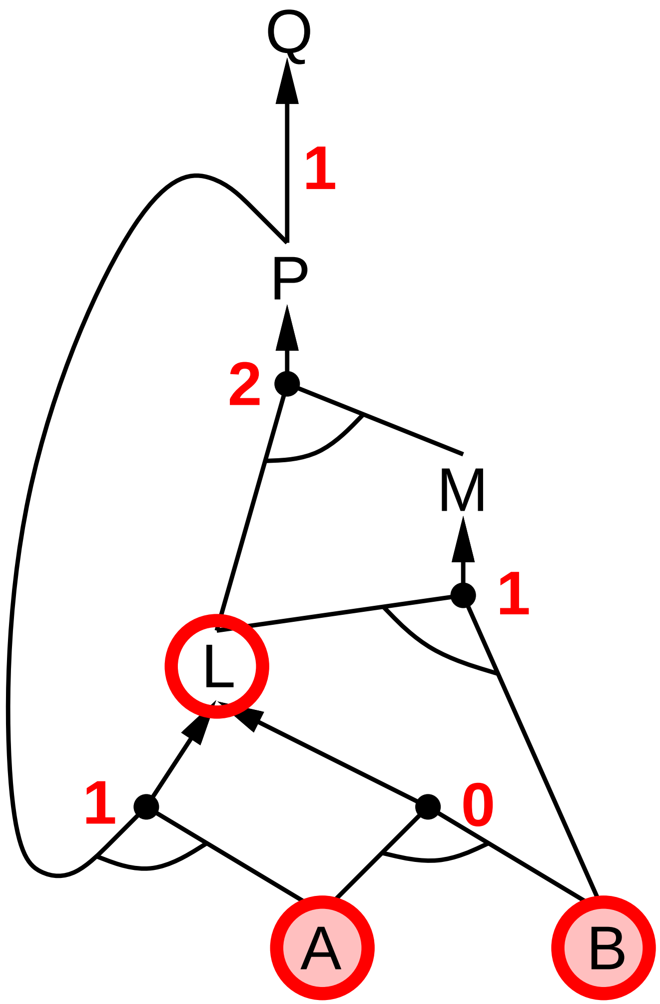
Forward chaining example



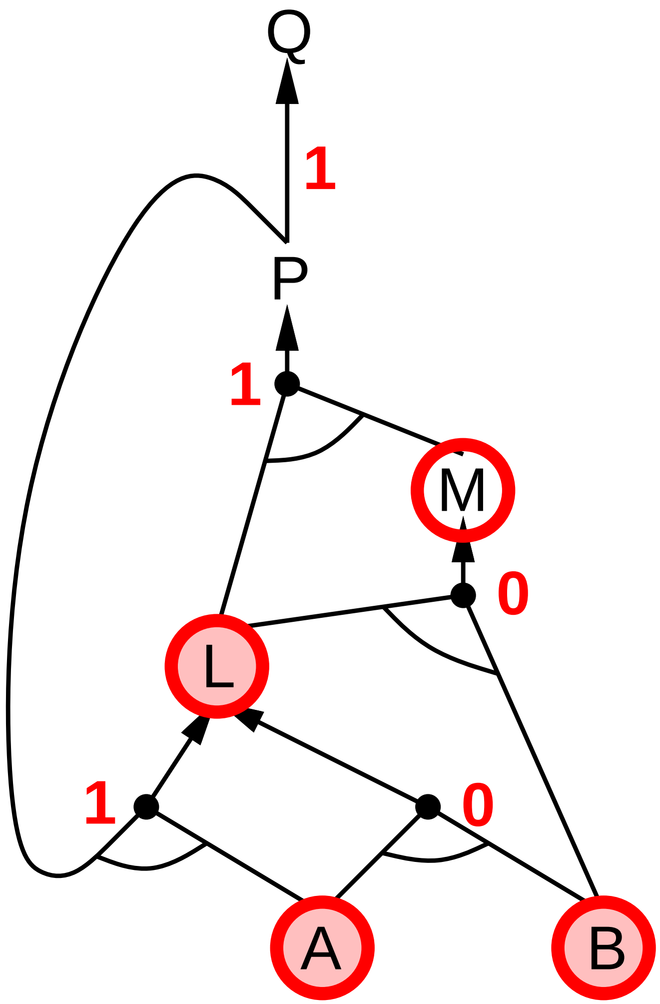
Forward chaining example



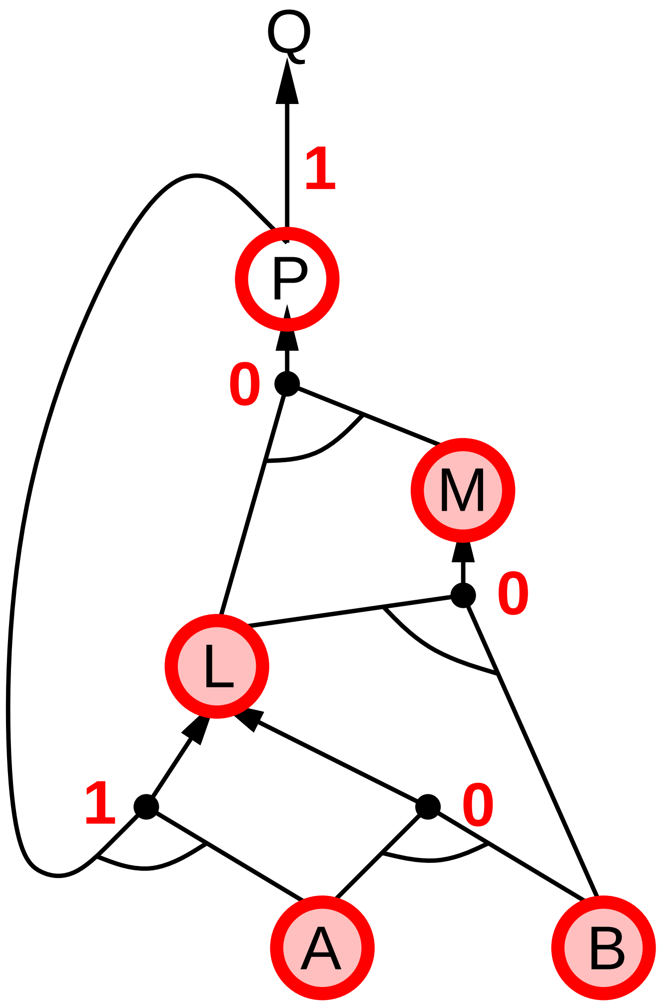
Forward chaining example



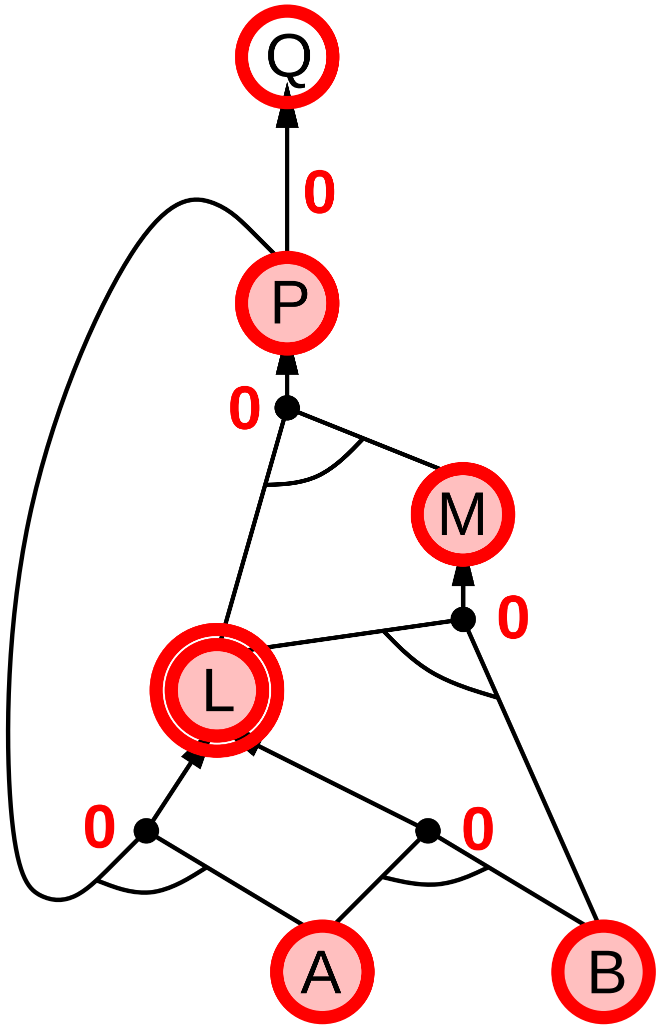
Forward chaining example



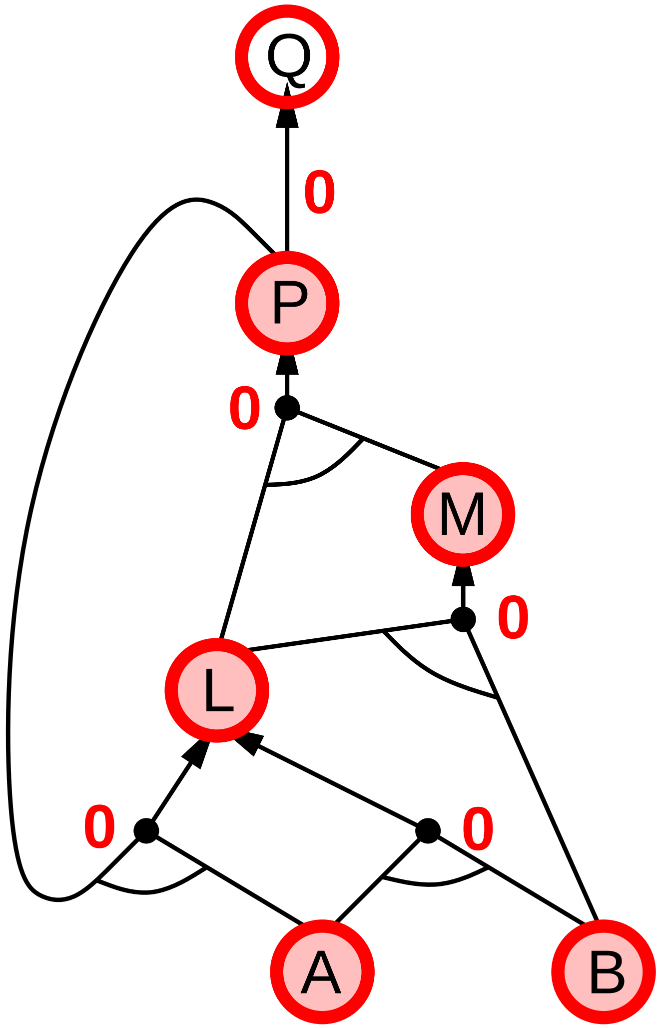
Forward chaining example



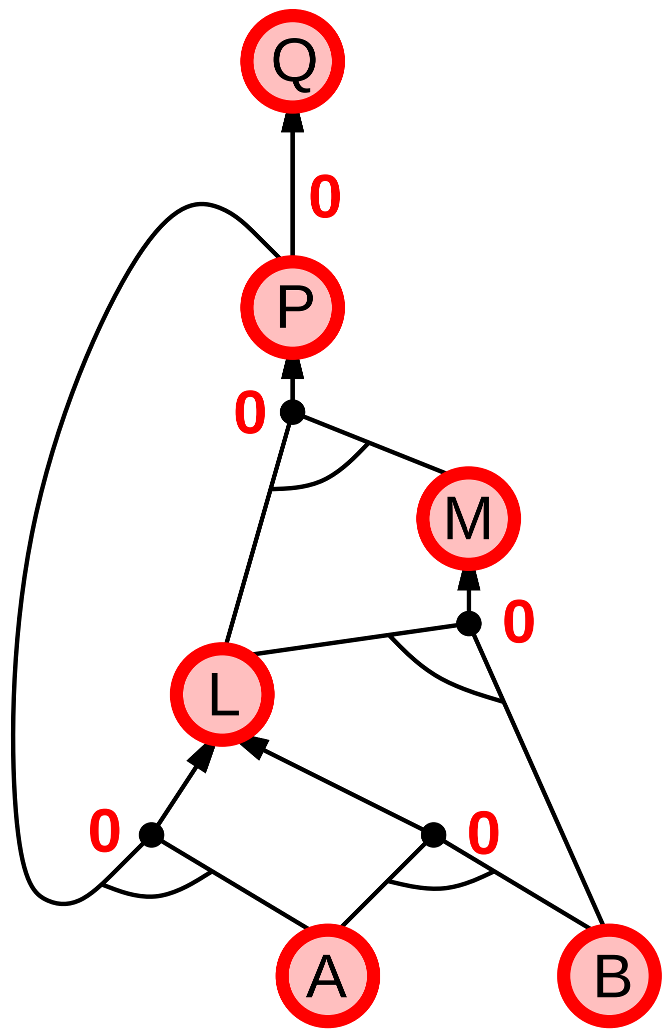
Forward chaining example



Forward chaining example



Forward chaining example



Proof of completeness

FC derives every atomic sentence that is entailed by KB

1. FC reaches a fixed point where no new atomic sentences are derived
2. Consider the final state as a model m, assigning true/false to symbols
3. Every clause in the original KB is true in m
   Proof: Suppose a clause $a_1 \wedge \ldots \wedge a_k \Rightarrow b$ is false in m
   Then $a_1 \wedge \ldots \wedge a_k$ is true in m and b is false in m
   Therefore the algorithm has not reached a fixed point!
4. Hence m is a model of KB
5. If $KB \models q$, q is true in **every** model of KB, including m

General idea: construct any model of KB by sound inference, check α

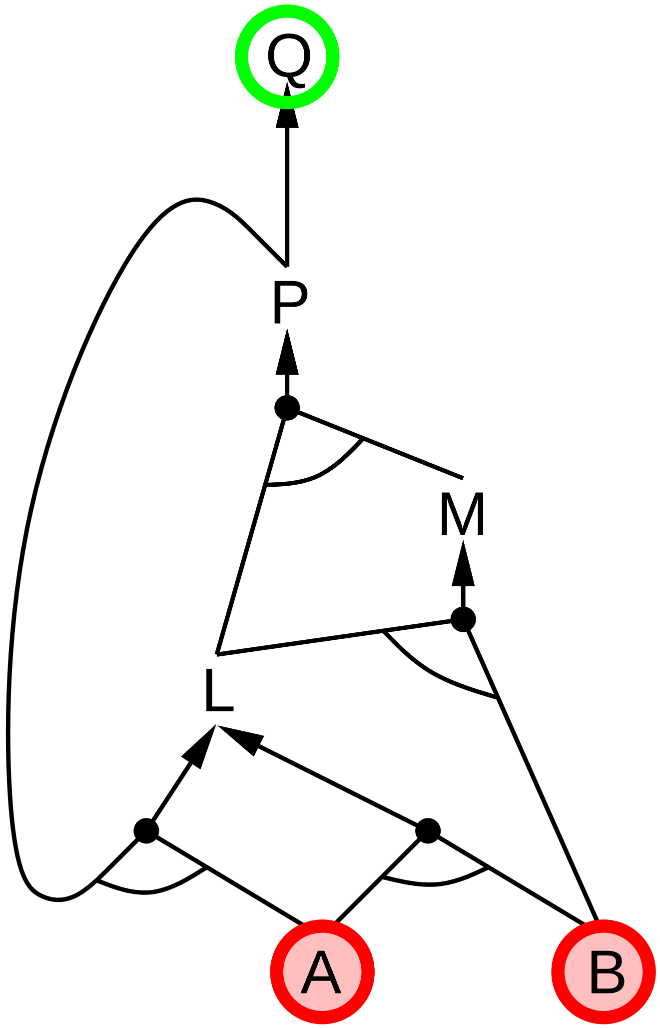
Backward chaining

Idea: work backwards from the query q:
to prove q by BC,
check if q is known already, or
prove by BC all premises of some rule concluding q

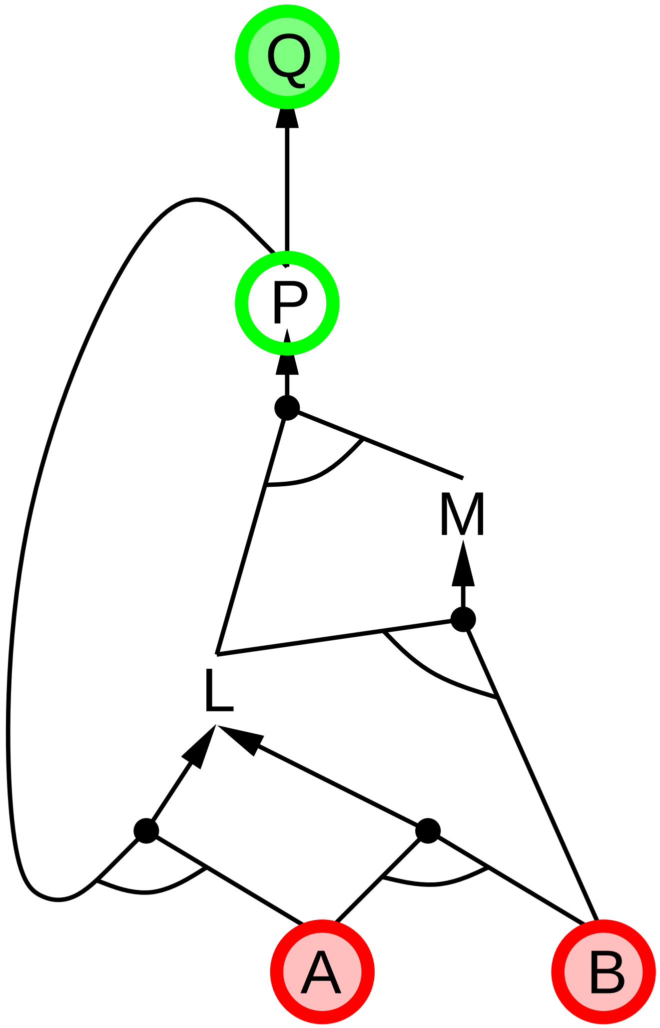
Avoid loops: check if new subgoal is already on the goal stack

Avoid repeated work: check if new subgoal
1) has already been proved true, or
2) has already failed

Backward chaining example



Backward chaining example



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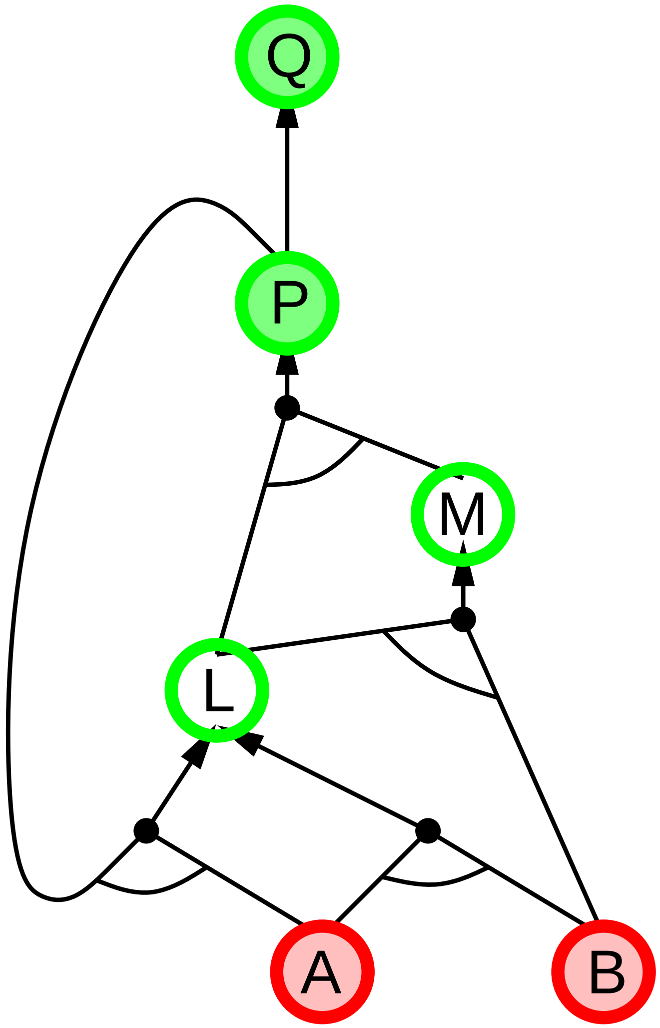
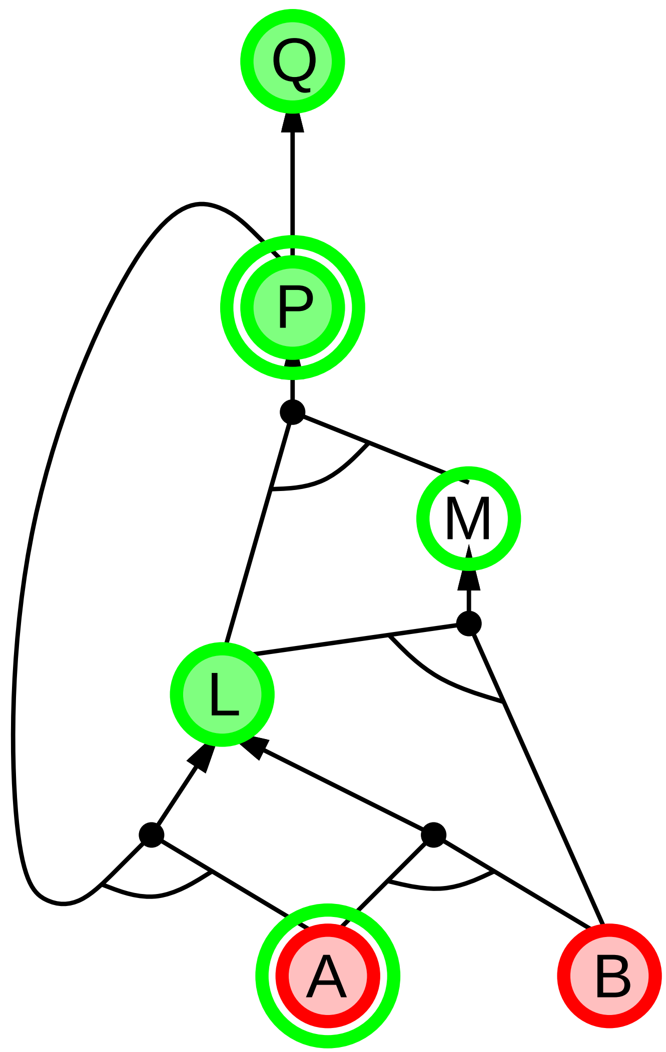


Backward chaining example



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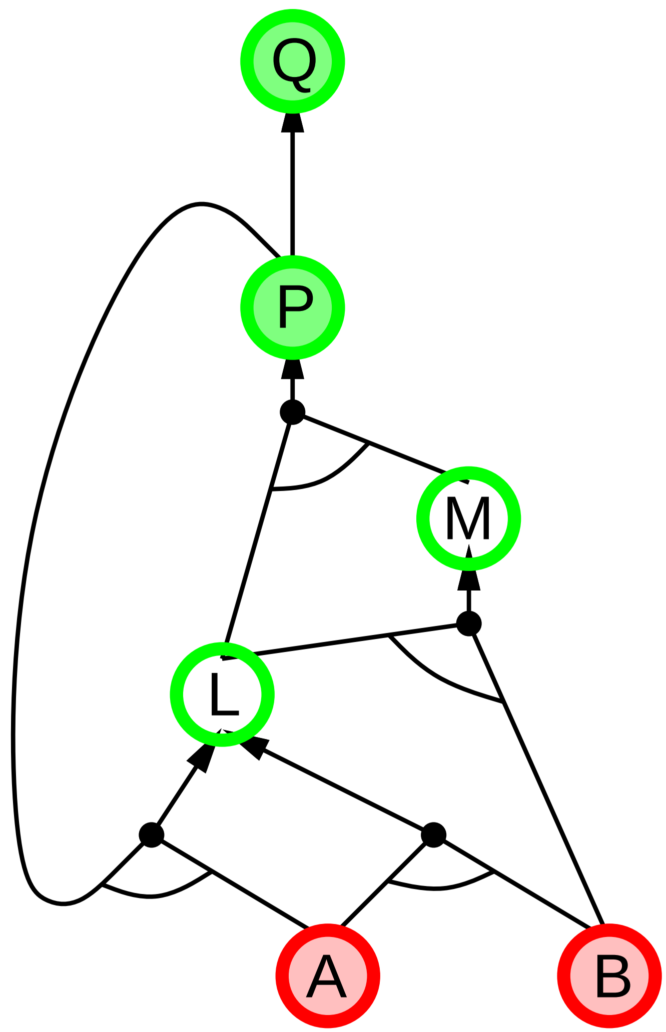
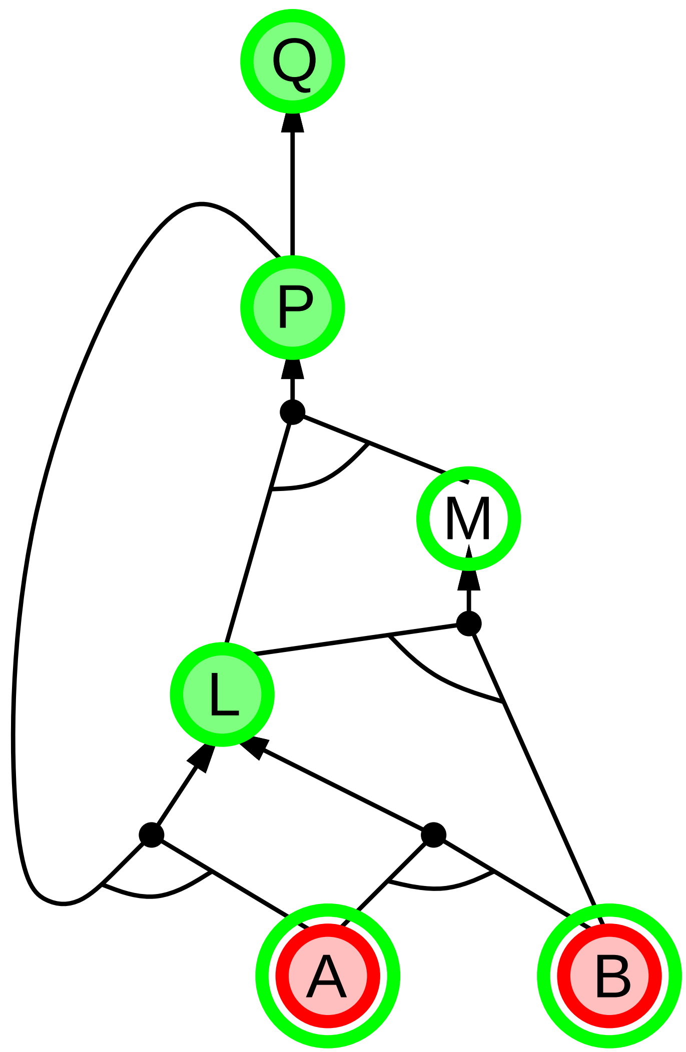
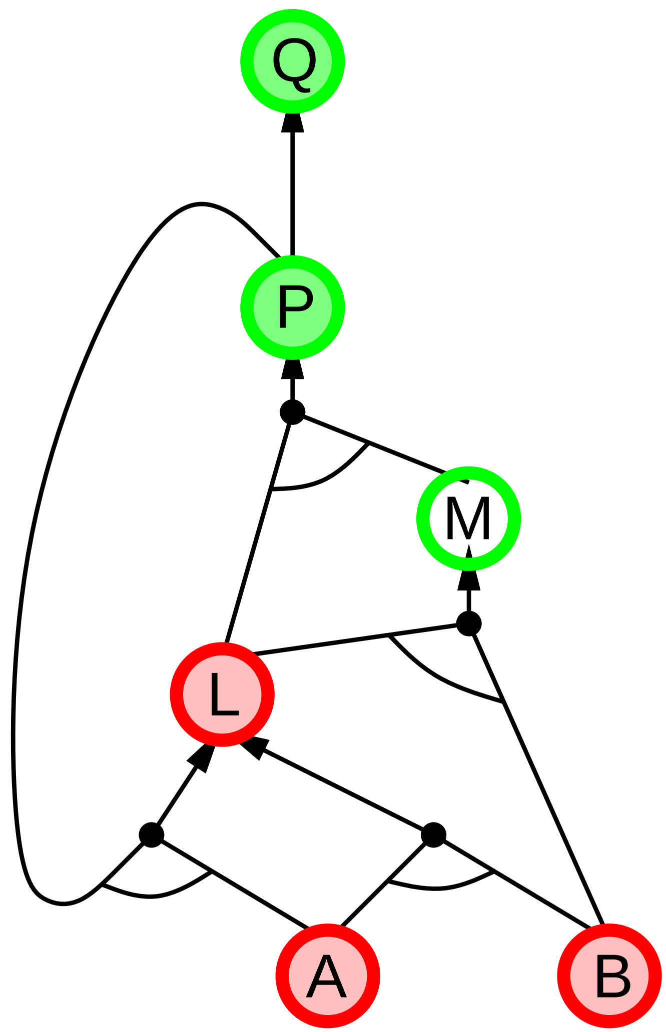


Backward chaining example



Backward chaining example



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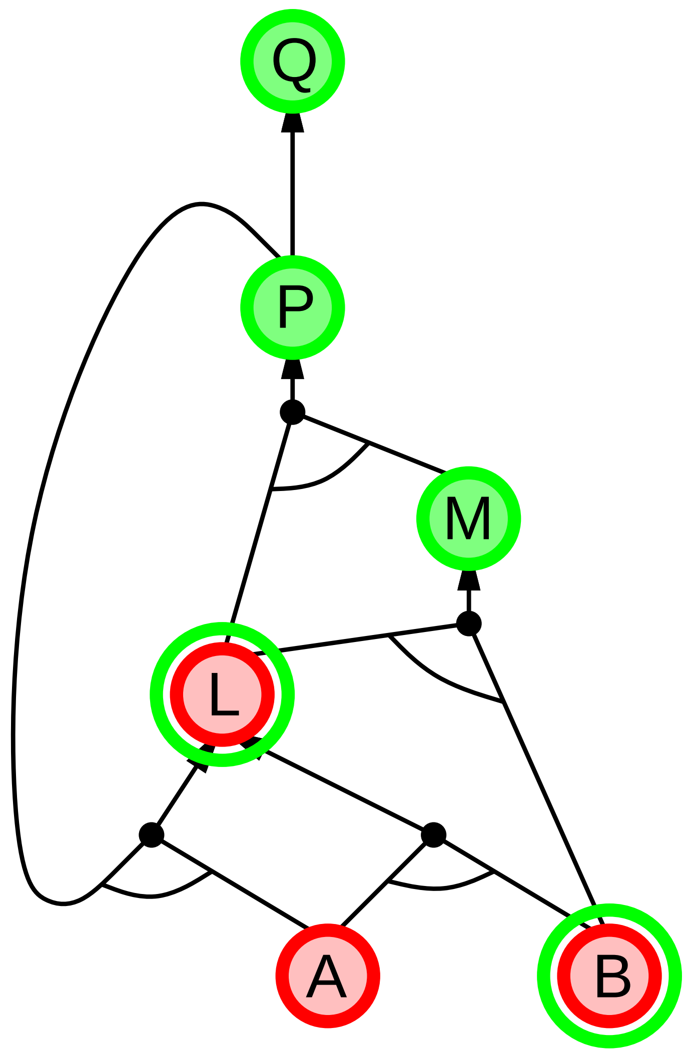

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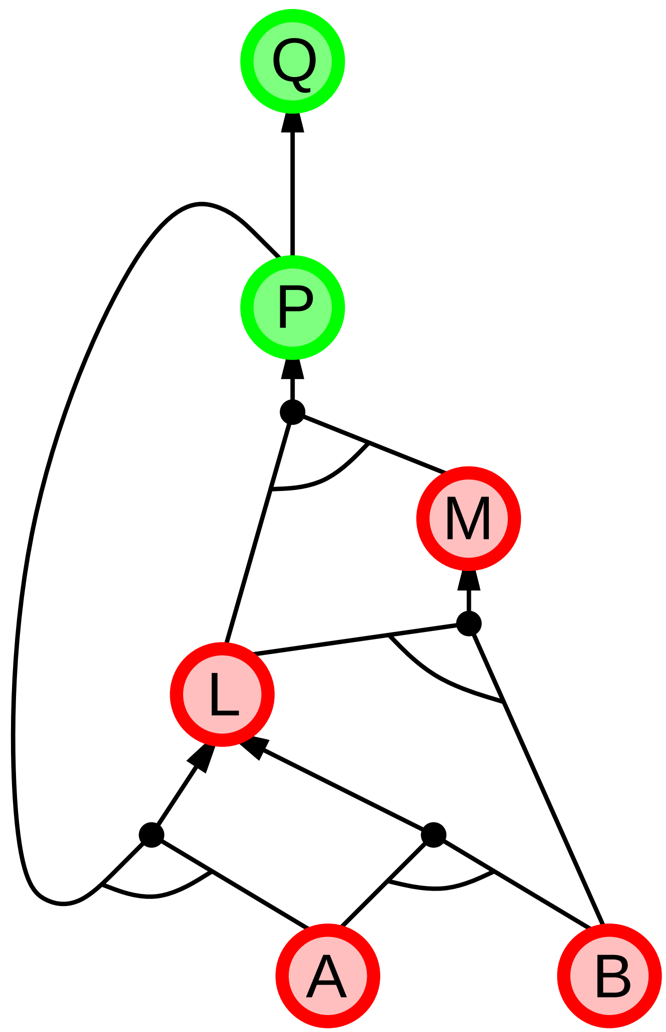

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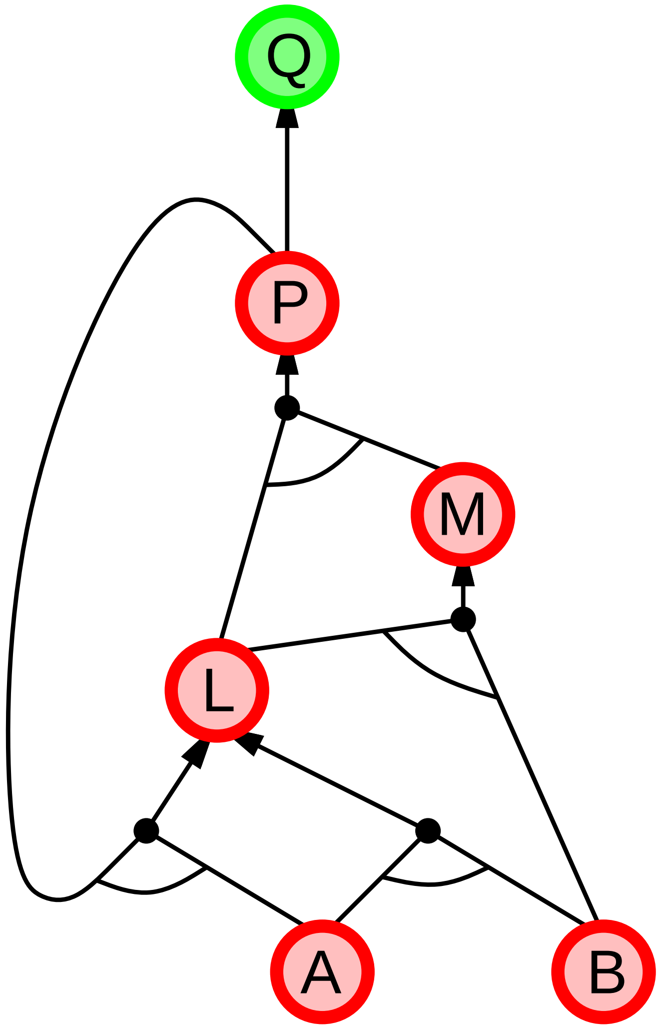

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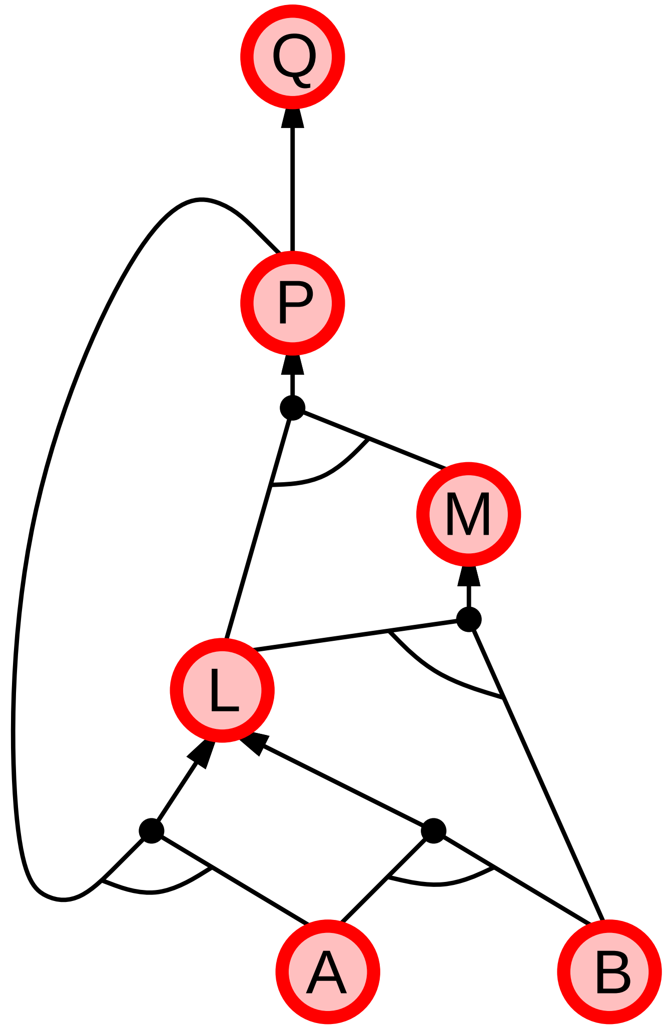


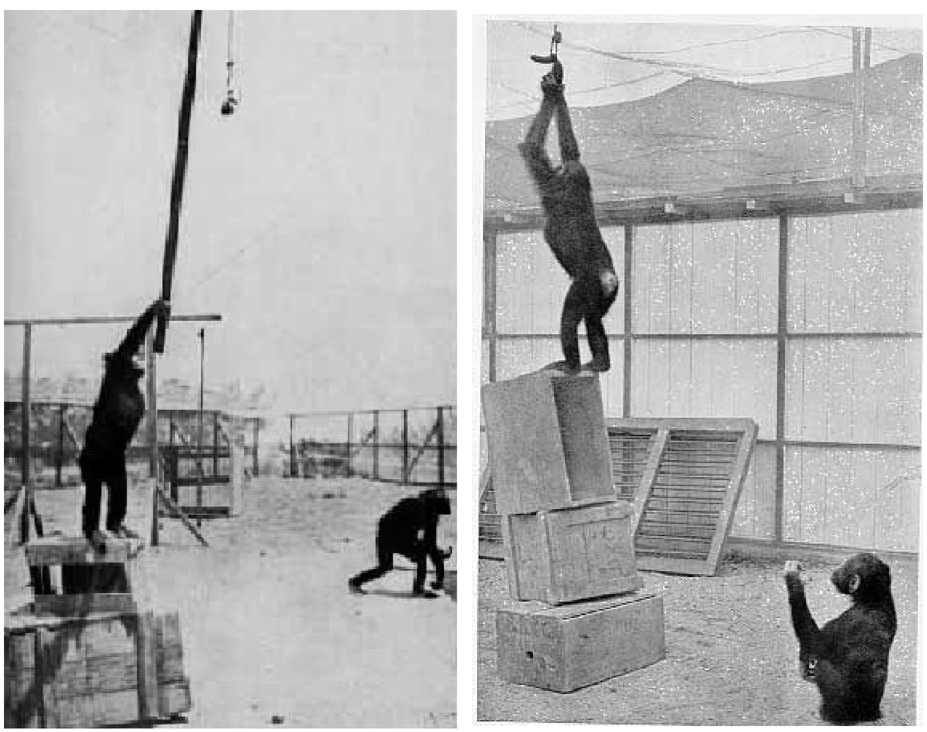
Forward vs. backward chaining

FC is data-driven, cf. automatic, unconscious processing,
e.g., object recognition, routine decisions

May do lots of work that is irrelevant to the goal

BC is goal-driven, appropriate for problem-solving,
e.g., Where are my keys? How do I get into a PhD program?

Complexity of BC can be **much less** than linear in size of KB



Summary

Logical agents apply inference to a knowledge base
to derive new information and make decisions

Basic concepts of logic:

- syntax: formal structure of sentences
- semantics: truth of sentences wrt models
- entailment: necessary truth of one sentence given another
- inference: deriving sentences from other sentences
- soundess: derivations produce only entailed sentences
- completeness: derivations can produce all entailed sentences

Wumpus world requires the ability to represent partial and negated information, reason by cases, etc.

Forward, backward chaining are linear-time, complete for Horn clauses
Resolution is complete for propositional logic

Propositional logic lacks expressive power