First-order logic

Chapter 8

Outline

◇ Why FOL?

◇ Syntax and semantics of FOL

◇ Fun with sentences

◇ Wumpus world in FOL

Pros and cons of propositional logic

- Propositional logic is **declarative**: pieces of syntax correspond to facts
  |--> knowledge and inference are separate. Inference is domain-independent.
- Propositional logic allows partial/disjunctive/negated information
  (unlike most data structures and databases)
- Propositional logic is **compositional**:
  meaning of $B_{1,1} \wedge P_{1,2}$ is derived from meaning of $B_{1,1}$ and of $P_{1,2}$
- Meaning in propositional logic is **context-independent**
  (unlike natural language, where meaning depends on context)
- Propositional logic has very limited expressive power
  (unlike natural language)
  E.g., cannot say "pits cause breezes in adjacent squares"
  except by writing one sentence for each square

Procedural approach: programs represent the computational processes. Data structures within programs can represent the facts. Lack any general mechanisms for deriving facts from other facts: each update to a data structure is done by a domain-specific procedure.

Natural languages are very expressive! but ambigious & context dependent.

First-order logic

Whereas propositional logic assumes world contains **facts**,
first-order logic (like natural language) assumes the world contains

- Objects: people, houses, numbers, theories, Ronald McDonald, colors, baseball games, wars, centuries . . .
- Relations: red, round, bogus, prime, multistoried . . .,
  brother of, bigger than, inside, part of, has color, occurred after, owns, comes between, . . .
- Functions: father of, best friend, third inning of, one more than, end of . . .

e.g. "one plus two equals three"
- objects?
- relations?
- functions?

Logics in general

Language	Ontological Commitment	Epistemological Commitment
Propositional logic	facts	true/false/unknown
First-order logic		
Temporal logic		
Probability theory		
Fuzzy logic	facts + degree of truth	known interval value

- Ontology often deals with questions concerning what entities exist or may be said to exist and how such entities may be grouped, related within a hierarchy, and subdivided according to similarities and differences. (wiki)

- Epistomology: the study or a theory of the nature and grounds of knowledge especially with reference to its limits and validity. (merriam-webster)

Logics in general

Language	Ontological Commitment	Epistemological Commitment
Propositional logic	facts	true/false/unknown
First-order logic	facts, objects, relations	true/false/unknown
Temporal logic	facts, objects, relations, times	true/false/unknown
Probability theory	facts	degree of belief
Fuzzy logic	facts + degree of truth	known interval value

Syntax of FOL: Basic elements

Constants	$KingJohn,\ 2,\ UCB, \ldots$
Predicates	$Brother,\ >, \ldots$
Functions	$Sqrt,\ LeftLegOf, \ldots$
Variables	$x,\ y,\ a,\ b, \ldots$
Connectives	$\land \quad \lor \quad \neg \quad \Rightarrow \quad \Leftrightarrow$
Equality	$=$
Quantifiers	$\forall\ \exists$

Syntax of FOL: Basic elements

$$
\begin{array}{rcl}
Sentence & \rightarrow & AtomicSentence \mid ComplexSentence \\
AtomicSentence & \rightarrow & Predicate \mid Predicate(Term, \ldots) \mid Term = Term \\
ComplexSentence & \rightarrow & (\ Sentence\ ) \mid [\ Sentence\ ] \\
& \mid & \neg\ Sentence \\
& \mid & Sentence \wedge Sentence \\
& \mid & Sentence \vee Sentence \\
& \mid & Sentence \ \Rightarrow\ Sentence \\
& \mid & Sentence \ \Leftrightarrow\ Sentence \\
& \mid & Quantifier\ Variable, \ldots\ Sentence \\
\\
Term & \rightarrow & Function(Term, \ldots) \\
& \mid & Constant \\
& \mid & Variable \\
\\
Quantifier & \rightarrow & \forall \mid \exists \\
Constant & \rightarrow & A \mid X_1 \mid John \mid \cdots \\
Variable & \rightarrow & a \mid x \mid s \mid \cdots \\
Predicate & \rightarrow & True \mid False \mid After \mid Loves \mid Raining \mid \cdots \\
Function & \rightarrow & Mother \mid LeftLeg \mid \cdots \\
\textsc{Operator Precedence} & : & \neg, =, \wedge, \vee, \Rightarrow, \Leftrightarrow
\end{array}
$$

Atomic sentences

Atomic sentence $= predicate(term_1, \ldots, term_n)$
or $term_1 = term_2$

Term $= function(term_1, \ldots, term_n)$
or $constant$ or $variable$

E.g., $Brother(KingJohn, RichardTheLionheart)$
$> (Length(LeftLegOf(Richard)), Length(LeftLegOf(KingJohn)))$

A term with no variables is called a ground term.

Complex sentences

Complex sentences are made from atomic sentences using connectives

$\neg S, \quad S_1 \wedge S_2, \quad S_1 \vee S_2, \quad S_1 \Rightarrow S_2, \quad S_1 \Leftrightarrow S_2$

E.g. $Sibling(KingJohn, Richard) \Rightarrow Sibling(Richard, KingJohn)$
$>(1,2) \vee \leq(1,2)$
$>(1,2) \wedge \neg{>}(1,2)$

The semantics must relate sentences to models in order to determine truth.

Truth in first-order logic

Sentences are true with respect to a model and an interpretation

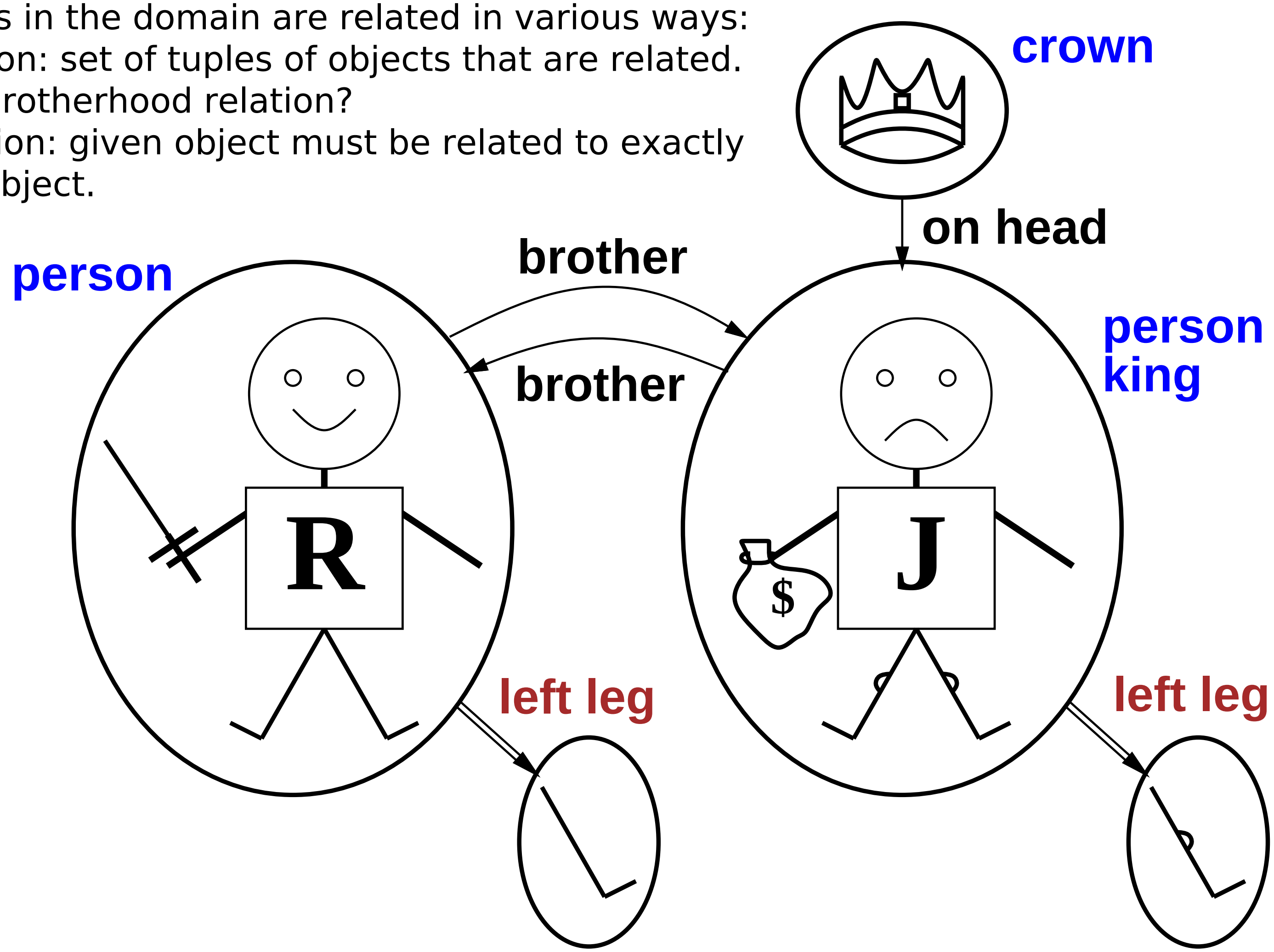
Model contains ≥ 1 objects (domain elements) and relations among them

Interpretation specifies referents for
constant symbols $\rightarrow$ objects
predicate symbols $\rightarrow$ relations
function symbols $\rightarrow$ functional relations

An atomic sentence $predicate(term_1, \ldots, term_n)$ is true
iff the objects referred to by $term_1, \ldots, term_n$
are in the relation referred to by $predicate$

Models for FOL: Example

Domain for FOL is the set of objects it contains.
Objects in the domain are related in various ways:
- relation: set of tuples of objects that are related.
  e.g. brotherhood relation?
- function: given object must be related to exactly
  one object.



Relation: the set of tuples of objects that are related.
Objects?
Relations? Unary relations, binary relations?
Functions?

Truth example

Consider the interpretation in which
$Richard \rightarrow$ Richard the Lionheart
$John \rightarrow$ the evil King John
$Brother \rightarrow$ the brotherhood relation

Under this interpretation, $Brother(Richard, John)$ is true
just in case Richard the Lionheart and the evil King John
are in the brotherhood relation in the model

Models for FOL: Lots!

Entailment in propositional logic can be computed by enumerating models

We **can** enumerate the FOL models for a given KB vocabulary:

For each number of domain elements n from 1 to ∞
  For each k-ary predicate P_k in the vocabulary
    For each possible k-ary relation on n objects
      For each constant symbol C in the vocabulary
        For each choice of referent for C from n objects . . .

Computing entailment by enumerating FOL models is not easy!

Universal quantification

$\forall \langle variables \rangle \; \langle sentence \rangle$

Everyone at Berkeley is smart:
$\forall x \;\; At(x, Berkeley) \;\Rightarrow\; Smart(x)$

$\forall x \;\; P$ is true in a model m iff P is true with x being **each** possible object in the model

Roughly speaking, equivalent to the conjunction of instantiations of P

$$
\begin{aligned}
& (At(KingJohn, Berkeley) \;\Rightarrow\; Smart(KingJohn)) \\
\wedge\; & (At(Richard, Berkeley) \;\Rightarrow\; Smart(Richard)) \\
\wedge\; & (At(Berkeley, Berkeley) \;\Rightarrow\; Smart(Berkeley)) \\
\wedge\; & \ldots
\end{aligned}
$$

Quiz

- What is the interpretation of

  $$\forall x \ At(x, Bogazici) \wedge Smart(x)$$

- Feedback:
  - https://tinyurl.com/yclbjjv6

A common mistake to avoid

Typically, $\Rightarrow$ is the main connective with $\forall$

Common mistake: using $\wedge$ as the main connective with $\forall$:

$$\forall x \;\; At(x, Berkeley) \wedge Smart(x)$$

means "Everyone is at Berkeley and everyone is smart"

Existential quantification

$\exists \langle variables \rangle \;\; \langle sentence \rangle$

Someone at Stanford is smart:
$\exists\, x \;\; At(x, Stanford) \wedge Smart(x)$

$\exists\, x \;\; P$ is true in a model m iff P is true with x being **some** possible object in the model

Roughly speaking, equivalent to the disjunction of instantiations of P

$$\begin{array}{ll} & (At(KingJohn, Stanford) \wedge Smart(KingJohn)) \\ \vee & (At(Richard, Stanford) \wedge Smart(Richard)) \\ \vee & (At(Stanford, Stanford) \wedge Smart(Stanford)) \\ \vee & \ldots \end{array}$$

Another common mistake to avoid

Typically, $\wedge$ is the main connective with $\exists$

Common mistake: using $\Rightarrow$ as the main connective with $\exists$:

$$\exists\, x \;\; At(x, Stanford) \;\Rightarrow\; Smart(x)$$

is true if there is anyone who is not at Stanford!

Properties of quantifiers

$\forall x\ \forall y$ is the same as $\forall y\ \forall x$

$\exists x\ \exists y$ is the same as $\exists y\ \exists x$

$\exists x\ \forall y$ is **not** the same as $\forall y\ \exists x$

$\exists x\ \forall y\ Loves(x, y)$

$\forall y\ \exists x\ Loves(x, y)$

Quantifier duality: each can be expressed using the other

$\forall x\ Likes(x, IceCream) \qquad \neg\exists x\ \neg Likes(x, IceCream)$

$\exists x\ Likes(x, Broccoli) \qquad \neg\forall x\ \neg Likes(x, Broccoli)$

Note: P(x,y) is interpreted as "x is a P of y".

Properties of quantifiers

$\forall x\ \ \forall y$ is the same as $\forall y\ \ \forall x$

$\exists x\ \ \exists y$ is the same as $\exists y\ \ \exists x$

$\exists x\ \ \forall y$ is **not** the same as $\forall y\ \ \exists x$

$\exists x\ \ \forall y\ \ Loves(x, y)$
"There is a person who loves everyone in the world"

$\forall y\ \ \exists x\ \ Loves(x, y)$
"Everyone in the world is loved by at least one person"

Quantifier duality: each can be expressed using the other

$\forall x\ \ Likes(x, IceCream) \qquad \neg\exists x\ \ \neg Likes(x, IceCream)$

$\exists x\ \ Likes(x, Broccoli) \qquad \neg\forall x\ \ \neg Likes(x, Broccoli)$

relate to De Morgan's rule?

Fun with sentences

Brothers are siblings

Fun with sentences

Brothers are siblings

$\forall\, x, y \;\; Brother(x, y) \;\Rightarrow\; Sibling(x, y)$.

"Sibling" is symmetric

Fun with sentences

Brothers are siblings

$\forall\, x, y \;\; Brother(x, y) \;\Rightarrow\; Sibling(x, y)$.

"Sibling" is symmetric

$\forall\, x, y \;\; Sibling(x, y) \;\Leftrightarrow\; Sibling(y, x)$.

One's mother is one's female parent

Fun with sentences

Brothers are siblings

$\forall\, x, y \;\; Brother(x, y) \;\Rightarrow\; Sibling(x, y)$.

"Sibling" is symmetric

$\forall\, x, y \;\; Sibling(x, y) \;\Leftrightarrow\; Sibling(y, x)$.

One's mother is one's female parent

$\forall\, x, y \;\; Mother(x, y) \;\Leftrightarrow\; (Female(x) \land Parent(x, y))$.

A first cousin is a child of a parent's sibling

Fun with sentences

Brothers are siblings

$\forall\, x, y \;\; Brother(x, y) \;\Rightarrow\; Sibling(x, y)$.

"Sibling" is symmetric

$\forall\, x, y \;\; Sibling(x, y) \;\Leftrightarrow\; Sibling(y, x)$.

One's mother is one's female parent

$\forall\, x, y \;\; Mother(x, y) \;\Leftrightarrow\; (Female(x) \land Parent(x, y))$.

A first cousin is a child of a parent's sibling

$\forall\, x, y \;\; FirstCousin(x, y) \;\Leftrightarrow\; \exists\, p, ps \;\; Parent(p, x) \land Sibling(ps, p) \land Parent(ps, y)$

Fun with sentences i.e. Kinship domain

$$\forall m, c \;\; Mother(c) = m \;\Leftrightarrow\; Female(m) \land Parent(m, c) \, .$$

One's husband is one's male spouse:

$$\forall w, h \;\; Husband(h, w) \;\Leftrightarrow\; Male(h) \land Spouse(h, w) \, .$$

Male and female are disjoint categories:

$$\forall x \;\; Male(x) \;\Leftrightarrow\; \neg Female(x) \, .$$

Parent and child are inverse relations:

$$\forall p, c \;\; Parent(p, c) \;\Leftrightarrow\; Child(c, p) \, .$$

A grandparent is a parent of one's parent:

$$\forall g, c \;\; Grandparent(g, c) \;\Leftrightarrow\; \exists p \;\; Parent(g, p) \land Parent(p, c) \, .$$

A sibling is another child of one's parents:

$$\forall x, y \;\; Sibling(x, y) \;\Leftrightarrow\; x \neq y \land \exists p \;\; Parent(p, x) \land Parent(p, y) \, .$$

Definitions "bottom-out" at a basic set of predicates.
What is the basic set here?